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15 NOVEMBER 1963

USER'S MANUAL for Interplanetary Error Propagation Program

CONTRACT NAS 5-3342

NASA

prepared for

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

PHILCO | WDL DIVISION

A SUBSIDIARY OF Ford Motor Company | PALO ALTO, CALIFORNIA

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Prepared by

PHILCO CORPORATION
A Subsidiary of Ford Motor Company
WDL Division
Palo Alto, California

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ABSTRACT 16809

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USER'S MANUAL FOR
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This report presents a description of the manner in which an error propagation data run is set up. A detailed description of the input data and the format to be used for the data are presented. The results obtained for an example test case using a nominal lunar trajectory are included.

Author

FOREWORD

This report is submitted to the National Aeronautics and Space Administration, Goddard Space Flight Center, in fulfilling the requirements of Contract NAS 5-3342.

The documentation provided by Philco WDL in support of the Interplanetary Error Propagation Program consists of the following three volumes:

- WDL-TR2184, "Programmer's Manual for Interplanetary Error Propagation Program"
- WDL-TR2185, "User's Manual for Interplanetary Error Propagation Program"
- Guidance and Control System Engineering Department Technical Report No. 4, "The Application of State Space Methods to Navigation Problems," by Stanley F. Schmidt

These volumes discuss the theory of the Schmidt-Kalman filter used in the program for data smoothing, the manner in which the program is used, and subroutine description and listing.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SUBROUTINES
USED IN ERROR PROPAGATION PROGRAM

Subroutine	Description
ARKTNS	Single Precision Arctangent
ARKTAN	Double Precision Arctangent
ASINH(X)	Function Evaluation
BODY	Calculates Accelerations Due to Perturbing Bodies
BVEC	Calculates B Vector
CHNGP	Determines when to Shift Body Center
COMPHQ	Computations for ONBTR and MONBTR Subroutines
CONSTI	Array of Input Constants
CONVPI	Converts Input Covariance Matrix to 1950
CORRTP	Updates P Matrix
CROSS	Cross Product
(CSH)S	FORTRAN II Card Image Input Subroutine
DOT	Function Forming Dot Product
DE6FN	FAP Integration Subroutine
EARTR	Updates Covariance Matrix for Earth Based Tracking
ECLIP	Transforms Coordinates through Transformations
ENCKE	Calculates Perturbation Due to Deviation from Conic
ERP	Prints Out Ephemeris Error
ERPT	Prints Out Time of Ephemeris Error
FINP	Data Input Subroutine
FNORM	Norm of a Vector
GHA	Greenwich Hour Angle
GOTOB	Main Subroutine for Integration of Trajectories
GOTOR	Iterates to Solve Kepler's Equation
GUID	Performs Guidance Calculations
HOUR	Read Printer Clock
HPHT	Performs Matrix Multiplication H*P*HT
INPUT	Converts Inputs to Equinox of 1950 Reference
INTR	FAP Ephemeris Subroutine

LIST OF SUBROUTINES (Continued)

Subroutine	Description
INVAO	Forms Inverse of Transition Matrix
INV3	Inverts up to a Six by Six Matrix
LOADO	Obtains Transition Matrix from T Array
LOADT	Puts Unit ICS on Perturbation Equations
MASS	Arranges Gravitational Constants of Bodies Considered
MATRX	Multiplies A*B=C or A*B*AT=C Maximum Dimension (10,10)
MATSUB	Error Propagation Logic Subroutine
MNA	Transformation to Selenocentric Coordinates
MNAND	Transformation for Selenocentric Velocities
MONBTR	Updates Covariance Matrix for Moon Beacons
MULT	Multiplies Two Three by Three Matrices
NUTAIT	Calculates Nutation Matrix
OBLN	Calculates Acceleration Due to Oblateness
ONBTR	Updates Covariance Matrix for Onboard Tracking
ORTC	Outputs Orbital Parameters
OUTC	Outputs Trajectory
OUTDAT	Outputs Calendar Date
OUTP	Outputs RMS Values of Orbital Parameters
PTRAN	Transforms P Matrix
RETRO	Performs Retro Fire
ROTATE	Calculates Transformation for Rotation About an Axis
ROTEQ	Calculates Matrix from Equinox 1950 to Mean Equinox of Date
RVIN	Transforms Coordinates from Spherical to Cartesian
RVOUT	Transforms Coordinates from Cartesian to Spherical
SDEC	Second Derivative Subroutine
SETN	Set Read and Write Tape Numbers
SHIFTP	Shifts Body Center
STEPC	Move Along Conic in Time
TIMEC	Converts Calendar Date to Days from 1950
TIMED	Converts Days Hours.Min Sec to Seconds
TRAC	Tracking Station Coordinates
TRANSH	Transforms H Matrix from Date to 1950

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USER'S MANUAL FOR
INTERPLANETARY ERROR PROPAGATION PROGRAM

SECTION 1

PROGRAM DESCRIPTION

1.1 INTRODUCTION

One of the challenging problems of space flight is the definition of a navigation system which has sufficient accuracy to achieve the objectives of the mission and at the same time has a system configuration and components which provide high reliability. The problem is complicated by the fact that there are a large number of parameters, all of which influence the accuracy. Furthermore, the parameters are coupled in such a manner that it is difficult to isolate individual effects.

In order to gain some insight into the type of problems which must be investigated, it is convenient to define navigation of a vehicle as the following subtasks:

a. Measurements of the observables (such as range rate, azimuth, elevation, line of sight, or distance to a terminal point).
b. Data smoothing to determine the current position and velocity.
c. The computation necessary for prediction of future states. (The determination of unperturbed course.)
d. Application of guidance law. (The determination of the control action required to arrive at the target in the desired manner.)
e. The application of control effort to carry out the guidance law.

With this definition of subtasks, we can point out some problems associated with them. As an example, for subtask a. one question is: How often should measurements be made and what type should be made to provide a given accuracy of position and velocity? It is obvious that this is strongly dependent on the type of smoothing utilized in obtaining the position and velocity from the measurements.

the true coordinates and those of the reference orbit, i.e. (initially at osculation x, y, z = 0)

$$x = X - X_E \qquad \begin{matrix} X \rightarrow Y, Z \\ x \rightarrow y, z \end{matrix}$$

The differential accelerations between actual and reference orbit positions of the vehicle are

$$\ddot{x} = \underbrace{\mu \left(- \frac{X}{R^3} + \frac{X_E}{R^3} \right)}_{\substack{\text{Encke} \\ \text{Acceleration}}} + \underbrace{f_X(X,Y,Z,t)}_{\substack{\text{Perturbation} \\ \text{Acceleration}}} \qquad \begin{matrix} X \rightarrow Y, Z \\ x \rightarrow y, z \end{matrix} \tag{3}$$

The quantities X_E, Y_E, Z_E are computed along a two-body reference conic. If the true orbit differs too greatly from the reference orbit, a new "osculating" reference orbit is taken as a new starting point. This process is called rectification and sets x, y, z = 0. The test which is made in the program to determine when rectification is required is the following.

$$\frac{(x^2 + y^2 + z^2)^{\frac{1}{2}}}{(X_E^2 + Y_E^2 + Z_E^2)^{\frac{1}{2}}} \geq .03$$

Maintaining this ratio small ensures the validity of the series expansion used in the computation of the Encke acceleration term. The program implementation of the above procedures for obtaining the nominal trajectory is shown in Figure 1-1.

1.3 TRANSITION MATRIX

The transition matrix is obtained from six sets of linear differential equations. These differential equations are obtained by making a Taylor series expansion about the nominal trajectory defined by Equation (2) and neglecting second-order and higher-order terms. Equation (2) is

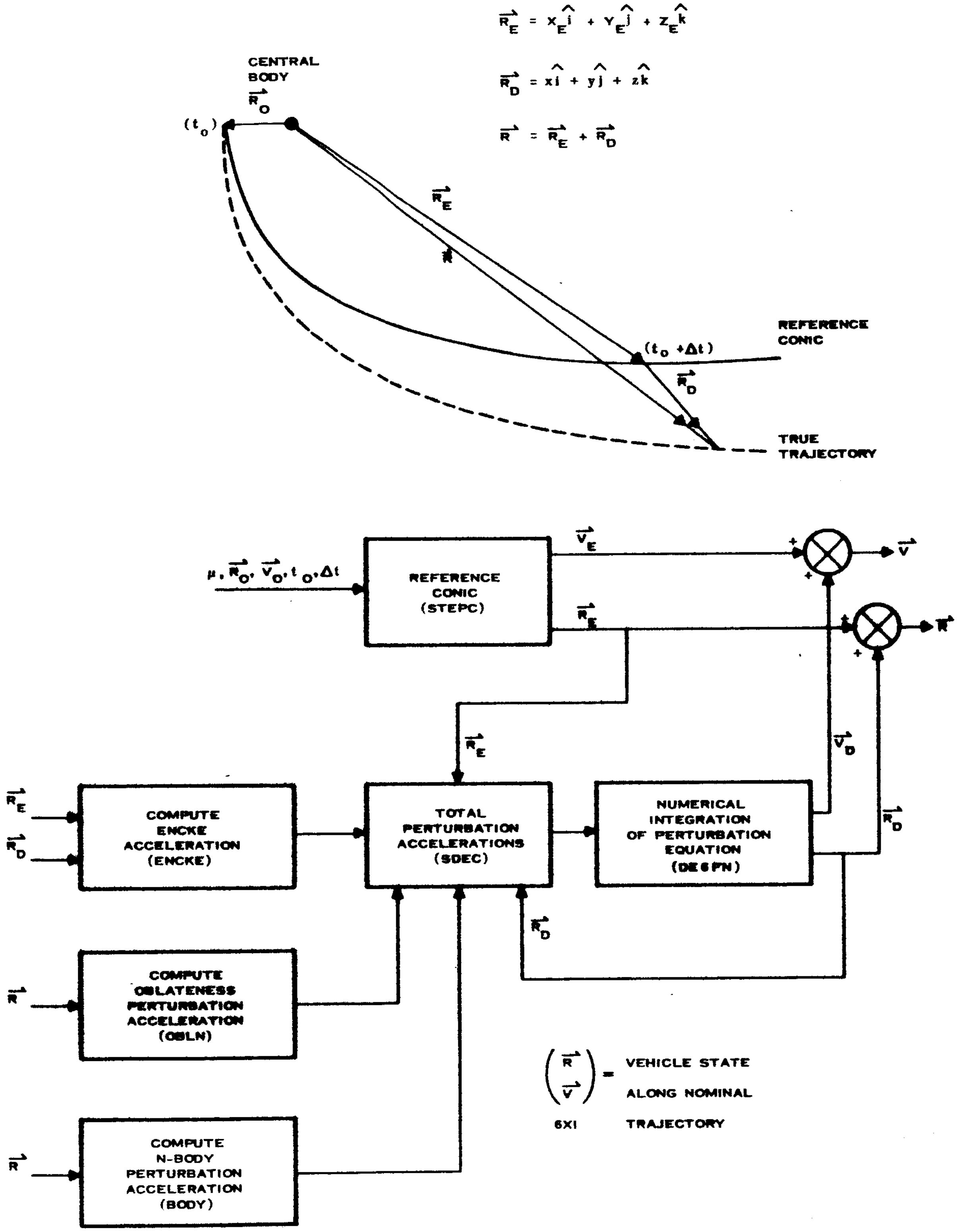

Figure 1-1 Generation of Nominal Trajectory

$$\ddot{X} = -\mu \frac{X}{R^3} + f_X(X,Y,Z,t) \qquad X \rightarrow Y,\ Z \tag{2}$$

For ease of presentation, the following relationships will be used in the remainder of this section.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \overleftarrow{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \quad \overleftarrow{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Then Equation (2) becomes

$$\ddot{X}_i = -\mu \frac{X_i}{R^3} + f_X\ (X_1,X_2,X_3,t) = f_i\ (\bar{X},t) \quad i = 1,2,3 \tag{2a}$$

Equation (2a) may then be expanded into a Taylor series as follows

$$\ddot{X}_i + \ddot{x}_i = f_i\ (\overleftarrow{X} + \overleftarrow{x},\ t)$$

$$\ddot{X}_i + \ddot{x}_i = f_i\ (\overleftarrow{X},t) + \sum_{j=1}^{3} \frac{\partial f_i}{\partial X_j} x_j + \frac{1}{2} \sum_{k=1}^{3} \sum_{j=1}^{3} \frac{\partial^2 f_i}{\partial X_k X_j} x_j\ x_k$$

Neglecting all terms other than the first order terms, the variational acceleration can be written as

$$\ddot{x}_i = \sum_{j=1}^{3} \frac{\partial f_i}{\partial X_j} x_j \qquad i = 1,2,3 \tag{4}$$

or

$$\underset{3x1}{\ddot{\overleftarrow{x}}} = \underset{3x3}{F(t)}\ \underset{3x1}{\overleftarrow{x}} \tag{5}$$

Defining the velocity states as:

$$\dot{x}_1 = x_4 \quad \dot{x}_2 = x_5 \quad \dot{x}_3 = x_6$$

permits Equation (5) to be written in first order form as

$$\underset{6x1}{\dot{\vec{x}}(t)} = \underset{6x6}{G(t)}\ \underset{6x1}{\vec{x}(t)} \tag{6}$$

where

$$G(t) = \underset{6x6}{\begin{bmatrix} 0 & I \\ F(t) & 0 \end{bmatrix}}$$

and

$$\vec{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

The solution of Equation (6) may be written as

$$\underset{6x1}{\vec{x}(t)} = \underset{6x6}{\Phi\,(t;t_o)}\ \underset{6x1}{\vec{x}(t_o)}$$

Where Φ is the transition matrix which transforms the state at time (t_o) to the state at (t). The Φ matrix is obtained by integrating six sets of the linear differential equations described by Equation (6). The solution of each set of linear equations yields one column of the transition matrix. Each column of the transition matrix describes the sensitivity of the states at time (t) to a deviation in a state at time (t_o).

$$\Phi(t;t_o) = \begin{bmatrix} \frac{\partial x_1(t)}{\partial x_1(t_o)} & \frac{\partial x_1(t)}{\partial x_2(t_o)} & \cdots & \frac{\partial x_1(t)}{\partial x_6(t_o)} \\ \frac{\partial x_2(t)}{\partial x_1(t_o)} & \vdots & & \vdots \\ \frac{\partial x_3(t)}{\partial x_1(t_o)} & \vdots & & \vdots \\ \frac{\partial x_6(t)}{\partial x_1(t_o)} & \frac{\partial x_6(t)}{\partial x_2(t_o)} & \cdots & \frac{\partial x_6(t)}{\partial x_6(t_o)} \end{bmatrix}$$

The generation of the transition matrix is accomplished in the program as shown in Figure 1-2. The integrators shown in the figure are actually part of the integration subroutine DE6FN. The matrix of the first variations of the acceleration function with respect to the positions

$$\frac{\partial f_i(\bar{X},t)}{\partial X_j} \qquad \begin{matrix} i = 1,3 \\ j = 1,3 \end{matrix}$$

is obtained from subroutine BODY for the central and n-body perturbations, and from OBLN for the oblateness perturbation. The partials are evaluated along the nominal trajectory.

Table 1-1 presents a comparison of one column of a transition matrix obtained from variational equations and the corresponding sensitivities obtained from taking the difference between a nominal trajectory and an integrated perturbed trajectory. The nominal trajectory was the one used in Section 4. The time period of the transition is one day.

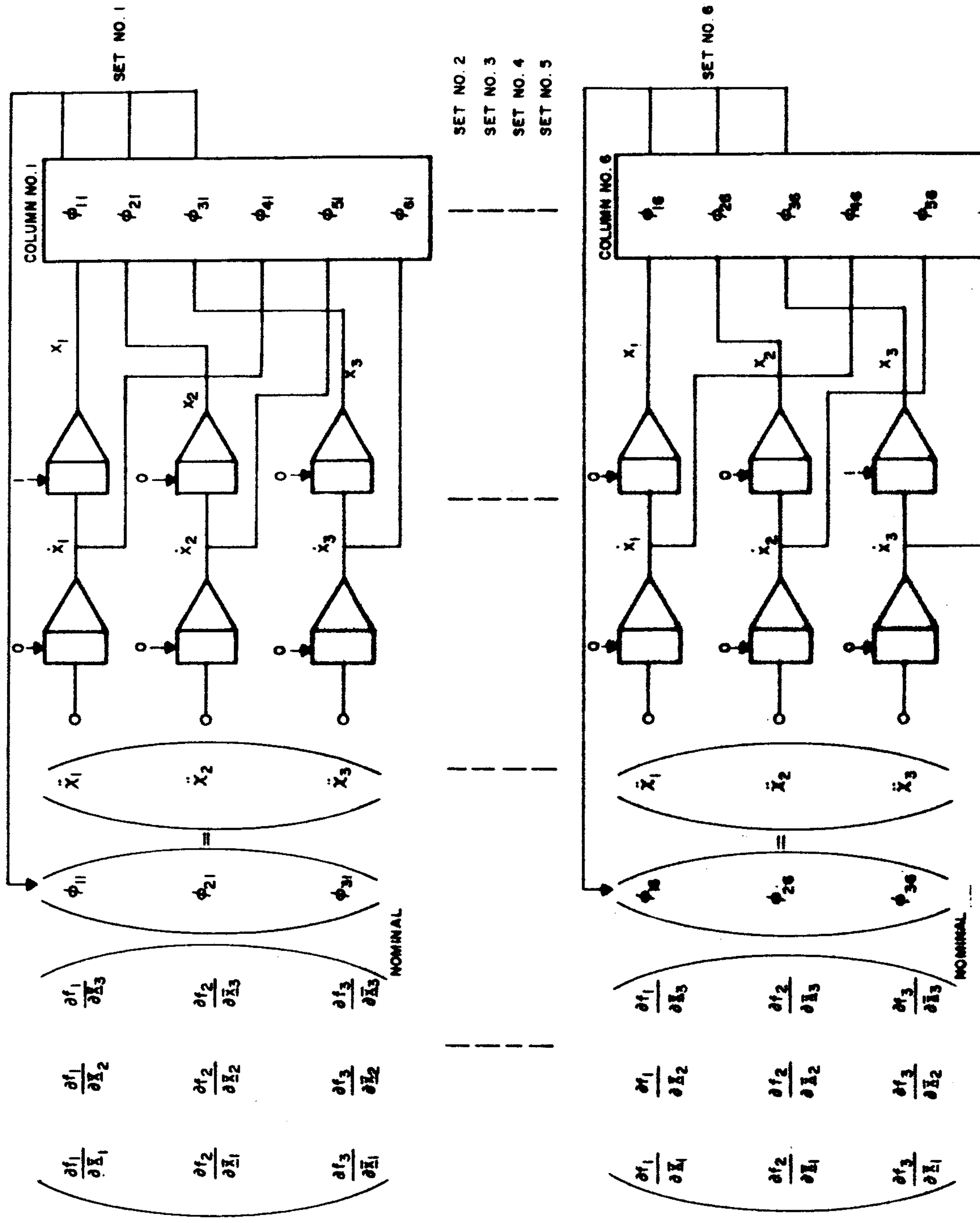

Figure 1-2 Generation of Transition Matrix

TABLE 1-1

COMPARISON OF SENSITIVITY COEFFICIENTS
OBTAINED BY TWO TECHNIQUES

Column No. 1 Transition Matrix	Difference between Nominal and Nominal with $x_1(0)$ Perturbed
$\frac{\partial x_1(t)}{\partial x_1(0)} = 13.58$	12.61
$\frac{\partial x_2(t)}{\partial x_1(0)} = 226.8$	277.4
$\frac{\partial x_3(t)}{\partial x_1(0)} = 121.1$	121.4
$\frac{\partial x_4(t)}{\partial x_1(0)} = -.780 \cdot 10^{-3}$	$-.794 \cdot 10^{-3}$
$\frac{\partial x_5(t)}{\partial x_1(0)} = -.332 \cdot 10^{-2}$	$-.332 \cdot 10^{-2}$
$\frac{\partial x_6(t)}{\partial x_1(0)} = .178 \cdot 10^{-2}$	$.178 \cdot 10^{-2}$

1.4 PROPAGATION AND UPDATING OF THE COVARIANCE MATRICES

The knowledge of state and deviation from the nominal, covariance matrices are propagated in time along the nominal trajectory using the transition matrix.[1]

$$\underset{6\times6}{P(t)} = \underset{6\times6}{\Phi(t;t_o)}\ \underset{6\times6}{P(t_o)}\ \underset{6\times6}{\Phi^T(t;t_o)}$$

$$\underset{6\times6}{PAR(t)} = \underset{6\times6}{\Phi(t;t_o)}\ \underset{6\times6}{PAR(t_o)}\ \underset{6\times6}{\Phi^T(t;t_o)}$$

where

P = the covariance matrix of knowledge of the state

PAR = the covariance matrix of deviations from the nominal

$\Phi(t;t_o)$ = the transition matrix from time (t_o) to time (t)

T = transpose

The updating of the state covariance matrix for the observations being made is performed in the following manner. It is assummed Schmidt-Kalman filter was used on the data for smoothing.

$$\underset{6\times6}{P_A(t)} = \underset{6\times6}{P_B} - \underset{6\times6}{P_B}\ \underset{6\times1}{H^T}\ \underset{\text{scalar}}{(HP_BH^T + Q)^{-1}}\ \underset{1\times6}{H}\ \underset{6\times6}{P_B}$$

where

P_A = the covariance matrix of the knowledge of state after an observation

P_B = the covariance matrix of the knowledge of state before the observation

---

(1) A general description of error propagation using transition matrices and updating using the Schmidt-Kalman filtering technique is presented in Philco WDL Guidance and Control Engineering Technical Report No. 4, "The Application of State Space Methods to Navigation Problems," Stanley F. Schmidt.

H = a matrix of partials of the measurement with respect to the state, i.e.:

$$H = \left(\frac{\partial MEAS}{\partial X} \quad \frac{\partial MEAS}{\partial Y} \quad \frac{\partial MEAS}{\partial Z} \quad \frac{\partial MEAS}{\partial \dot{X}} \quad \frac{\partial MEAS}{\partial \dot{Y}} \quad \frac{\partial MEAS}{\partial \dot{Z}} \right)$$

Q = the error in measurement

The updating of the covariance matrix of deviations from the nominal for a guidance correction is described in the subroutine writeup[2] of subroutine GUID. The result of a correction is the following updating.

$$\underset{6x6}{PAR_A} = \underset{6x6}{\begin{bmatrix} I & 0 \\ -A_2^{-1}A_1 & 0 \end{bmatrix}} \underset{6x6}{\begin{bmatrix} PAR_B - P_B \end{bmatrix}} \underset{6x6}{\begin{bmatrix} I & 0 \\ -A_2^{-1}A_1 & 0 \end{bmatrix}^T} + \underset{6x6}{P_B} + \underset{6x6}{\begin{bmatrix} 0 & 0 \\ 0 & E(qq^T) \end{bmatrix}}$$

where

PAR_A = the covariance matrix of deviations from the nominal after a correction

PAR_B = the covariance matrix of deviations from the nominal before the correction

P_B = the covariance matrix of the knowledge of the state before the guidance correction

$A_2^{-1}A_1$ = transition matrices obtained in the application of a guidance law (see GUID writeup)

$E(qq^T)$= the covariance matrix of the error in making the guidance correction

The covariance matrix of knowledge is updated at the time of a guidance correction in the following manner.

$$\underset{6x6}{P_A} = \underset{6x6}{P_B} + \alpha \underset{6x6}{\begin{bmatrix} 0 & 0 \\ 0 & E(qq^T) \end{bmatrix}}$$

(2) Programmer's Manual for Interplanetary Error Propagation Program

where

> α = the ability to make an onboard observation of the quality of the guidance correction

The subroutines used in performing the above described functions are shown in Figure 1-3. Of particular note is that these processes are controlled by subroutine MATSUB. This will be described in more detail later, but MATSUB must be called at frequent enough intervals so that covariance matrices can be propagated and updated for any measurements being made, or guidance corrections which are performed. MATSUB also must be called if output is desired because it calls the output subroutines.

1.5 OUTPUT DATA FORMATS

The output of the error propagation program displays at each output time the following data.

a. Date, time and Cartesian state along the trajectory
b. Orbital elements
c. Selenographic data if moon centered
d. RMS values of the knowledge of the Cartesian state and the orbital elements
e. Normalized state covariance matrix in N, V, W coordinates
f. Condition of the knowledge of state covariance matrix
g. Geometry at the tracking stations or geometry relative to the moon beacon or celestial bodies making measurements at the time
h. RMS knowledge of the position and velocity following the observations (1950)
i. Guidance data if it is a guidance run.

To facilitate identification of the output quantities, a lettered code precedes the floating point representation of the quantity. Three samples of the error propagation program output are presented in Figures 1-4, 1-5 and 1-6.

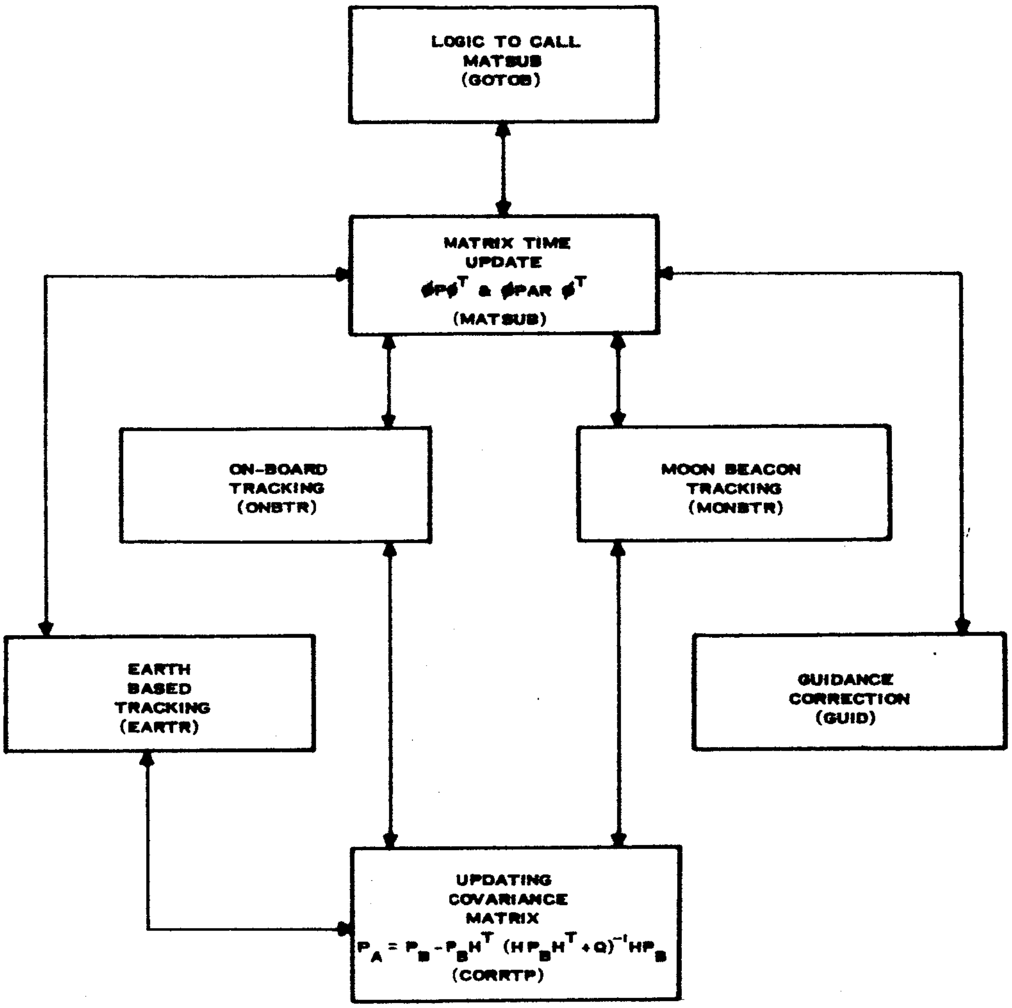

Figure 1-3 Subroutines used in Updating Covariance Matrices

```
   EARTH  CENTERED
  YEAR=1965  MONTH= 1  DAY= 4  HOUR=18  MIN=21  SEC=25.328
    2 DAYS   13 HRS.  20 MIN.  -0.    SEC.        DATE     JULIAN DATE 2438765.26487648                EQUATORIAL COORDINATES
    X 0.23613530E 06     Y-0.25159840E 06     Z-0.13649198E 06   DX 0.72870085E 00   DY-0.56756561E 00   DZ-0.30305443E-00
    R 0.37106831E 06   DEC-0.21582170E 02    RA 0.31318410E 03    V 0.97209959E 00  PTH 0.80960217E 02   AZ 0.69354603E 02
  SMA 0.33122230E 06   ECC 0.98776035E 00   INC 0.29521152E 02  LAN 0.35749587E 03  APF 0.13965501E 03  RCA 0.40540445E 04
  RTE 0.37106831E 06   LAT-0.21582170E 02   LON 0.29368451E 03   VE 0.25037215E 02  PTE 0.21974844E 01  AZE 0.90123142E 02
                                                 RMS VALUES BEFORE OBSERVATIONS
    X 0.15380076E-00     Y 0.50238660E 00     Z 0.25616293E-00   DX 0.12379506E-04   DY 0.13122203E-04   DZ 0.12912937E-04
  SMA 0.65397485E 01   ECC 0.21071508E-05   INC 0.31785559E-02  LAN 0.72681425E-02  APF 0.62672215E-02  RCA 0.70122673E 00
                                                     RMS N,V,W COORDINATES
                          RMS POSITION, KM                                          RMS VELOCITY KM/SEC
        NORMAL                VELOCITY              MOMENTUM             NORVEL              VELVEL              MOMENVEL
    0.44691277E-00        0.33739770E-00        0.16762517E-00        0.13582218E-04      0.11842080E-04      0.12941283E-04
                                 NORMALIZED STATE COVARIANCE MATRIX IN  N,V,W  COORDINATES
    0.09999999E 01        0.85499323E 00        0.33492908E-00        0.34923969E-00      0.13317284E-00      0.31849396E-01
                          0.09999999E 01        0.31981097E-00        0.19093522E-00      0.30150268E-00      0.30291345E-01
                                                0.09999999E 01        0.76198900E-01      0.49440572E-01      0.42130416E-00
                                                                      0.09999999E 01      0.39996518E-01      0.12426223E-01
                                                                                          0.09999999E 01      0.77048504E-02
                                                                                                              0.09999999E 01
  CONDITION OF STATE COVARIANCE MATRIX
  DETERMINATE OF MATRIX = 0.37492514E-15    PRODUCT OF DIAGONAL = 0.17238355E-14  RATIO PROD.DIAG./DET. = 0.45978126E 01
  TRACKER STATION GOSTRC
  RNG 0.36939311E 06  RGR 0.68699031E 00  AZM 0.13181247E 03  AZR 0.30306121E-02  ELE 0.14643694E 02  ELR 0.25440844E-02
  CELESTIAL BODY  1
  RNG 0.37106832E 06  RGR 0.                RA-0.47027707E 02  RAR 0.              DEC 0.21637844E 02  DCR 0.
  CELESTIAL BODY  2
  RNG 0.48132655E 05  RGR 0.                RA 0.81229971E 02  RAR 0.              DEC-0.22675100E 02  DCR 0.
  CELESTIAL BODY  3
  RNG 0.14676994E 09  RGR 0.                RA-0.74857161E 02  RAR 0.              DEC-0.22703966E 02  DCR 0.
  KNOWLEDGE OF STATE AFTER ALL OBSERVATIONS
           RMS POSITION= 0.57830021E 00            RMS VELOCITY= 0.21875157E-04

  GUID DATA FOLLOWS
     RMS FTA TARGET MISS= 0.43526523E 01    RMS POS DEV FROM NOM= 0.23307793E 03
       RMS KNOW. OF MISS= 0.18468852E 01    RMS VEL DEV FROM NOM= 0.71570611E-02
            RMS VEL REQ= 0.79361197E-04
```

Figure 1-4 Output Format

```
 MOON  CENTERED
YEAR=1965  MONTH= 1  DAY= 5  HOUR= 3  MIN= 1  SEC=25.327
  2 DAYS  22 HRS.  -0 MIN.   0.    SEC.       DATE   JULIAN DATE 2438765.62598759          EQUATORIAL COORDINATES
   X 0.39961447E 04    Y 0.68192083E 04    Z 0.29660798E 04   DX-0.17852780E-00   DY-0.14229971E 01   DZ-0.60014850E 00
   R 0.84420615E 04  DEC 0.20569629E 02   RA 0.59629142E 02    V 0.15546611E 01  PTH-0.68332876E 02   AZ 0.80086669E 02
 SMA 0.39020718E 04  ECC 0.14926171E 01  INC 0.15725979E 03  LAN 0.17607598E 03  APF 0.22148480E 03  RCA 0.19222273E 04
 RTE 0.40129530E 06  LAT-0.21406179E 02  LON 0.16369702E 03   VE 0.27422594E 02  PTE 0.22248760E 01  AZE 0.89962333E 02
 SELENOGRAPHIC LON=   0.28952353E 03 LAT=  -0.48067360E-00
 SMA 0.38214429E 04  ECC 0.15345196E 01  INC 0.17947108E 03  LAN 0.22418487E 03  APF 0.39258117E 02  RCA 0.20426362E 04
                                         RMS VALUES BEFORE OBSERVATIONS
   X 0.19560365E-00    Y 0.45367756E-00    Z 0.20288750E-00   DX 0.13209674E-04   DY 0.22992780E-04   DZ 0.10392432E-04
 SMA 0.60427868E-01  ECC 0.36502510E-04  INC 0.17777029E-03  LAN 0.33999361E-02  APF 0.16125467E-02  RCA 0.14952853E-00
                                                RMS N,V,W COORDINATES
                       RMS POSITION, KM                                              RMS VELOCITY KM/SEC
       NORMAL               VELOCITY            MOMENTUM            NORVEL               VELVEL             MOMENVEL
   0.14161002E-00      0.51115975E 00      0.62528327E-01      0.10400134E-04      0.26288985E-04      0.34485637E-05
                              NORMALIZED STATE COVARIANCE MATRIX IN  N,V,W  COORDINATES
   0.09999999E 01     -0.91065546E 00      0.66435834E-02      0.92655063E 00     -0.90496018E 00      0.86963179E-02
                       0.09999999E 01     -0.38514348E-02     -0.95279434E 00      0.97513814E 00     -0.12991357E-01
                                           0.09999999E 01      0.21065509E-02     -0.85541379E-02      0.43410732E-00
                                                               0.09999999E 01     -0.94115286E 00      0.88073954E-02
                                                                                   0.09999999E 01     -0.18125810E-01
                                                                                                       0.09999999E 01

CONDITION OF STATE COVARIANCE MATRIX
DETERMINATE OF MATRIX = 0.84907202E-20   PRODUCT OF DIAGONAL = 0.32296893E-14  RATIO PROD.DIAG./DET. = 0.38037872E 06
TRACKER STATION WOMERA
RNG 0.39557113E 06  RGR 0.89420582E 00  AZM 0.73917066E 02  AZR-0.42284096E-02  ELE 0.63680377E 02  ELR 0.34992847E-02
CELESTIAL BODY  1
RNG 0.40129531E 06  RGR 0.                RA-0.46658890E 02  RAR 0.              DEC 0.21462239E 02  DCR 0.
CELESTIAL BODY  2
RNG 0.84420618E 04  RGR 0.                RA 0.59414640E 02  RAR 0.              DEC-0.20527933E 02  DCR 0.
CELESTIAL BODY  3
RNG 0.14674317E 09  RGR 0.                RA-0.74465893E 02  RAR 0.              DEC-0.22665174E 02  DCR 0.
KNOWLEDGE OF STATE AFTER ALL OBSERVATIONS
          RMS POSITION= 0.48047671E-00             RMS VELOCITY= 0.25864284E-04

GUID DATA FOLLOWS
   RMS FTA TARGET MISS= 0.43525964E 01    RMS POS DEV FROM NOM= 0.25724375E 02
     RMS KNOW. OF MISS= 0.96113975E 00    RMS VEL DEV FROM NOM= 0.58544870E-02
          RMS VEL REQ= 0.76156274E-03
```

Figure 1-5 Output Format

```
STATE COVARIANCE MATRIX AT END POINT
 0.32427592E-00   0.56143057E-01   0.57711668E-01   0.20218088E-03  -0.58771013E-04  -0.46955410E-04
 0.56143057E-01   0.12146325E-00  -0.12990589E-00   0.20599356E-04  -0.10251372E-03   0.16347326E-03
 0.57711668E-01  -0.12990589E-00   0.25554832E-00   0.53310609E-04   0.12344085E-03  -0.29467940E-03
 0.20218088E-03   0.20599356E-04   0.53310609E-04   0.12795607E-06  -0.24942876E-07  -0.50689401E-07

-0.58771013E-04  -0.10251372E-03   0.12344085E-03  -0.24942876E-07   0.91914206E-07  -0.15223651E-06
-0.46955410E-04   0.16347326E-03  -0.29467940E-03  -0.50689401E-07  -0.15223651E-06   0.34386954E-06

STATE TRANSITION MATRIX FROM INJECT TO END POINT
 0.91037010E 03  -0.53331733E 03  -0.28863958E 03   0.96430945E 06   0.78281710E 06   0.41036503E 06
 0.11878956E 04  -0.81103717E 03  -0.42964409E 03   0.12409330E 07   0.11614326E 07   0.60976345E 06
 0.56445710E 03  -0.37428603E 03  -0.22409976E 03   0.58992471E 06   0.55051097E 06   0.29137257E 06
 0.42093010E-00  -0.23057126E-00  -0.12592795E-00   0.44830022E 03   0.34241153E 03   0.17935441E 03
-0.57834594E 00   0.38580001E-00   0.19822198E-00  -0.60591896E 03  -0.55117447E 03  -0.28862767E 03
-0.27728929E-00   0.17710974E-00   0.12100135E-00  -0.29003306E 03  -0.26898827E 03  -0.14373595E 03

STATE COVARIANCE MATRIX AT INJECTION
 0.66555648E 02   0.62042827E 02   0.32676131E 02  -0.63680355E-01   0.42936660E-01   0.23701259E-01
 0.62042827E 02   0.58412839E 02   0.30767352E 02  -0.59422732E-01   0.40486512E-01   0.22321825E-01
 0.32676131E 02   0.30767352E 02   0.16215312E 02  -0.31296434E-01   0.21296512E-01   0.11819126E-01
-0.63680355E-01  -0.59422732E-01  -0.31296434E-01   0.60935586E-04  -0.41130362E-04  -0.22700198E-04
 0.42936660E-01   0.40486512E-01   0.21296512E-01  -0.41130362E-04   0.28165305E-04   0.15267725E-04
 0.23701259E-01   0.22321825E-01   0.11819126E-01  -0.22700198E-04   0.15267725E-04   0.89614989E-05

B*T,B*R,VINF PARTIALS AT END POINT PBV
 0.15977661E 01  -0.15629211E 01  -0.58619384E 00  -0.50336426E 04  -0.45399963E 04  -0.20144531E 04
 0.14786807E 01   0.10062155E-00  -0.17220672E 01  -0.24772520E 04  -0.10712937E 04   0.74732623E 03
 0.83702030E-03  -0.78963457E-03  -0.30612090E-03  -0.16080318E 01  -0.14514785E 01  -0.65281040E 00

COVARIANCE MATRIX OF KNOWLEGE OF B*T,B*R,VINF
    0.24237631E-00      0.25231545E-00     -0.92785759E-07
    0.25231545E-00      0.29241472E 01     -0.47596198E-06
   -0.92785759E-07     -0.47596198E-06      0.10234408E-09

COVARIANCE MATRIX OF GUIDANCE ERRORS
 0.30714864E 02   0.18769085E 02   0.87636875E 01   0.16340658E-01  -0.66288880E-02  -0.30172846E-03
 0.18769085E 02   0.19576881E 02   0.90859381E 01   0.76145343E-02   0.22923592E-02   0.58962551E-02
 0.87636875E 01   0.90859381E 01   0.53436772E 01   0.34802368E-02   0.20789096E-02   0.22426739E-02
 0.16340658E-01   0.76145343E-02   0.34802368E-02   0.13282887E-04  -0.78093420E-05  -0.25812083E-05
-0.66288880E-02   0.22923592E-02   0.20789096E-02  -0.78093420E-05   0.14732607E-04   0.12231600E-05
-0.30172846E-03   0.58962551E-02   0.22426739E-02  -0.25812083E-05   0.12231600E-05   0.19162976E-04

COVARIANCE MATRIX OF KNOWLEDGE OF TARGET MISS PARAMETERS
    0.32463957E-00      0.56493802E-01      0.57649583E-01
    0.56493802E-01      0.12188348E-00     -0.13012976E-00
    0.57649583E-01     -0.13012976E-00      0.25516876E-00

COVARIANCE MATRIX OF TARGET MISS PARAMETERS
    0.29276346E 02      0.15030643E 02      0.68009289E 01
    0.15030643E 02      0.98320358E 01      0.39657258E 01
    0.68009289E 01      0.39657258E 01      0.26531639E 01
```

Figure 1-6 Output Format

The following is a description of the symbols and meaning of the quantities used in Figure 1-4. The description will be performed on a line-by-line basis.

Line

1. Body center for the coordinate system

2. Calendar date

3a. Time from injection

3b. Equinox of output coordinate system (date or 1950)

3c. Julian date

3d. Coordinate system orientation (Earth's equatorial or ecliptic)

4. (X,Y,Z) Cartesian position and (DX,DY,DZ) Cartesian velocity in the body center and coordinates described by Lines 1, 3b and 3d

5. R = radius from central body

   DEC = declination of vehicle

   RA = right ascension of vehicle

   V = inertial velocity

   PTH = flight path angle

   AZ = velocity azimuth

6. SMA = semi-major axis

   ECC = eccentricity

   INC = orbital inclination

   LAN = longitude of the ascending node

   APF = argument of perifocus

   RCA = radius of closest approach

7. The quantities in this line refer to earth coordinates independent of the body center

   RTE = radius to earth

   LAT = latitude sub-satellite point on earth

   LON = longitude sub-satellite point on earth

V = inertial velocity
PTE = flight path angle
AZE = velocity azimuth

} Instantaneous inertial earth fixed coordinate's x-axis along Greenwich meridian

The quantities described above for the first seven lines of output are the same for all body centers. As can be seen in Figure 1-5, for the case of moon-centered output there are two additional lines of output. The first line describes the selenographic latitude and longitude of the sub-satellite point on the moon. The second line contains the orbital elements in the instantaneous inertial moon-fixed coordinates with the x-axis along the zero longitude meridian.

Following the trajectory data printout, the error data is presented. Under the title of "RMS Values before Observations," the following data is output.

<u>Line</u>

1. (X,Y,Z,DX,DY,DZ) RMS values of the knowledge of the Cartesian states prior to observations at this time

2. RMS values of the knowledge of the orbital elements

3. RMS N, V, W COORDINATES: These coordinates refer to the following directions relative to the orbit:

$$\left(\frac{\vec{v}}{|\vec{v}|} \times \frac{\vec{R} \times \vec{v}}{|R \times v|};\ \frac{\vec{v}}{|v|};\ \frac{\vec{R} \times \vec{v}}{|R \times v|} \right)$$

   Below Line 3, the RMS values of the knowledge of the states in the N, V, W coordinate system are printed out along with the normalized covariance matrix.

Below the normalized covariance matrix, the condition of the knowledge of the state covariance matrix, is printed out. The purpose of this is to warn the operator when the matrix becomes ill-conditioned. The next printout describes the measurement geometries which are being used at the time. The tracker station names, moon beacon number, and celestial body names which are being used at the time are printed out. The key to the printout associated with the measurement geometry is the following:

RNG = Range
RGR = Range rate
AZM = Azimuth
AZR = Azimuth rate
ELE = Elevation
ELR = Elevation Rate
RA = Right ascension
RAR = Right ascension rate
DEC = Declination
DCR = Declination rate

If any of the above quantities are printed out as zero, it indicates that the quantity was not computed. Following the tracker geometry data printout, the RMS knowledge of position and velocity (1950) following the observations are printed out.

If the run being performed is a guidance run, an additional section of output is presented which contains guidance data. The quantities which are printed out are the following.

RMS KNOW OF MISS = Knowledge of the target miss parameters
RMS VEL REQ = Velocity correction required at this time
RMS POS DEV FROM NOM = Position deviation from the nominal
RMS VEL DEV FROM NOM = Velocity deviation from the nominal

If the guidance law being used is fixed time of arrival, the following additional information is printed out.

RMS FTA MISS = Fixed time of arrival target miss

If the guidance law being used is constant energy WRT the target, the following data is printed in place of the FTA datum.

RMS TARGET POS MISS = B vector miss parameters

$$\left(\sqrt{(\Delta\vec{B} \cdot \hat{T})^2 + (\Delta\vec{B} \cdot \hat{R})^2}\right)$$

RMS VINFINITY MISS = Error in hyperbolic excess velocity

The final format of printout is presented in Figure 1-6. This is the data which is printed out at the completion of a run. The headings for the quantities in the printout describe what the matrices represent.

SECTION 2

INPUT DATA

2.1 DESCRIPTION OF INPUTS

The program is designed to accept two sets of input data cards for each run to be performed. Runs may be stacked by merely stacking the sets of input data cards. The two sets of data cards will be distinguished from one another by the routines which read the cards. The first set of cards is for the input subroutine FINP and will be called FINP cards. The second set of cards is read by GOTOB and will be called GOTOB cards. The set of GOTOB cards is composed of an arbitrary number of pairs of cards. The use of the GOTOB and FINP input data in the program is shown in a general flow diagram in Figure 2-1.

The input-output tapes which the program uses are set up by subroutine SETN (see subroutine writeup). The program is presently set up to use Tapes 2 and 3. If a change in either or both tape numbers is desired, subroutine SETN should be recompiled with the desired tape numbers.

2.2 FINP INPUT DATA

The FINP data cards essentially determine the type of data runs being made, the nominal trajectory, and type of output coordinate system desired. The quantities which may be input through FINP are presented in Table 2-1. The input order of quantities in the table is not important. The quantities which are input through FINP are stored in common and will be used on successive runs if the runs are stacked. The input data are not set to zero following the first run. Therefore, if the input data desired for a second run are different from the first, all the quantities to be changed must be input. This includes placing zeros in place of quantities which are no longer desired. The three exceptions to this rule are three quantities associated with guidance runs. The quantities GUILD, CIOMP and RETR are all set to zero following a run, and if the options are desired for the next run they must be input.

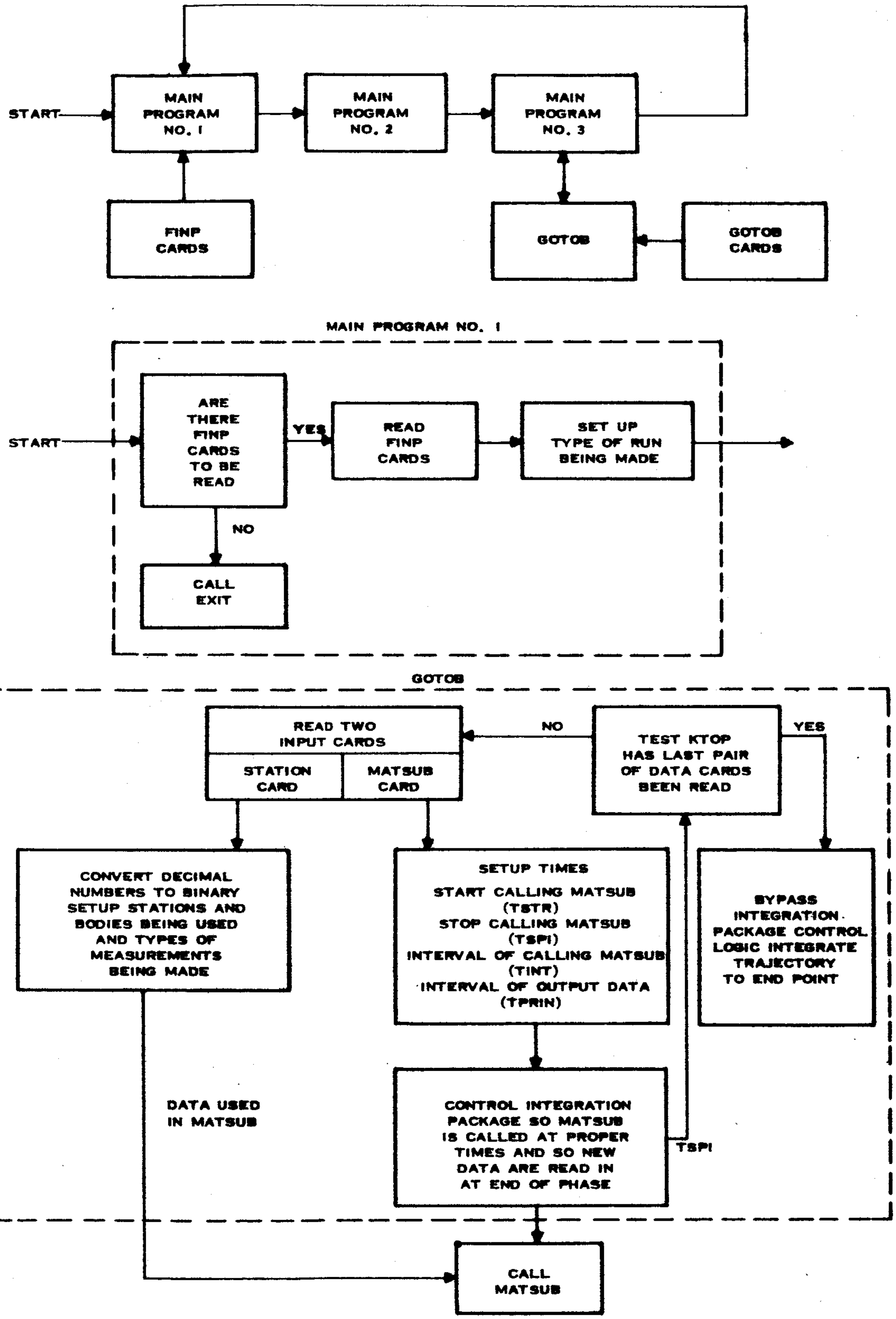

Figure 2-1 Use of FINP and GOTOB Input Data

2.3 FINP DATA CARD FORMAT

A detailed description of the card format and capabilities of FINP is presented in the subroutine FINP writeup in the Programmer's Manual. The data card is divided into four data fields as shown below.

	Conversion Code	Location	Value	Exponent
First Field	1	2-6	7-16	17-18
Second Field	19	20-24	25-34	35-36
Third Field	37	38-42	43-52	53-54
Fourth Field	55	56-60	61-70	71-72

The following information describes the format to be used in placing the data on the FINP cards.

a. Decimal Points. Decimal points may be placed anywhere in the value field except that they may not occur in the same column as minus signs (11 punch). If the decimal point would normally appear at the right of the number punched in the value field, then it is optional.

b. Minus Signs. Minus signs are 11 punches over any digit of the field. If all of the available columns of the field are not used, minus signs may be punched as the left character of the field.

c. Values. Values must always be written to the extreme left of a field. It is not necessary that the entire field be filled as the first blank denotes the end of value.

d. Location. The location may be specified by either a variable or array name. If the location is left blank, then the location counter within the routine is decreased by (1) and the associated

TABLE 2-1

FINP INPUT DATA

FINP	Description of Quantity	Description
S	S Common Array	Permits overlay of constants in S used to input measurement variances, etc.
TIM	Type of Inputs Coordinates	1. = Cartesian Equator of 1950 2. = Cartesian Equator of date 3. = Cartesian Earth fixed 4.5.6. Corresponds to 1., 2., 3. but spherical$^{(1)}$
CIBDY	Central Body	1. = Earth 3. = Sun 5. = Mars 2. = Moon 4. = Venus 6. = Jupiter
TIBDY	Target Body	1. = Earth 3. = Sun 5. = Mars 2. = Moon 4. = Venus 6. = Jupiter
SKTB	Type of Stop	1. = Impact, closest approach, time 2. = Closest approach, time 3. = Time
TSTOP	Stop Time	Format (DAYS HOURS . MIN SEC)$^{(6)}$
X	Initial Position	Dimension (3) Cartesian or Spherical$^{(1)}$
VX	Initial Velocity	Dimension (3) Cartesain or Spherical$^{(1)}$
DATE	Initial Date	Format (YR MONTH . DAY)
FDATE	Fractional Date	Format (HOUR MIN : Sec _ _)
OUTTP	Output Coordinates	0. = Equator date 1. = Equator 1950 2. = Eccliptic of date
PI	Initial Knowledge of State Covariance of Matrix	Dimension (21)$^{(2)}$
PUPIN	Type of Input Covariance Matrix (PI)	1. = Launch Pad$^{(2)}$ 3. = Date 2. = Injection 4. = 1950 5. = Use old transformation

Superscripts explained at end of Table.

TABLE 2-1 (Concluded)

FINP INPUT DATA

FINP	Description of Quantity	Description
GUID[3]	Type of Guidance Desired	0. = No guidance 1. = Fixed Time of Arrival[4] 2. = Constant energy WRT the target[4]
CIOMP[3]	Compute Guidance Partial Matrix	0. = Guidance matrix is being input 1. = Guidance matrix is to be computed
FTA[5]	Fixed Time Arrival Partial Matrix	Dimension (3,6) Fixed Time of Arrival Guidance partial matrix
CTE[5]	Constant Energy Partial Matrix	Dimension (3,6) constant energy WRT The target GUID partial matrix
PARI	Deviation from Nominal Covariance Matrix	Dimension (21)[2]
PARIN	Type of Input	1. = Launch Pad[2] 2. = Injection[2] 3. = Date 4. = 1950 5. = Use old transformation
TGUID	Times for Guidance Corrections	Dimension (6) Format (DAYS HOURS . MIN SEC)
GID	Number of Corrections	1. to 6.
RETR[3]	Retro Maneuver	0. = No retro 1. = Perform retro maneuver

SUPERSCRIPTS USED IN TABLE 2-1

(1) Spherical coordinates require specific order for input quantities (see subroutine writeup for RVIN).

(2) Input covariance matrices require specific order for input quantities (see subroutine writeup for CONVPI).

(3) Quantity is set to zero following each run and must be input for each run if the option is desired.

(4) Requires guidance partial matrix to be computed (CIOMP) or input (FTA or CTE).

(5) Matrix must be read in by columns.

(6) Formats for time inputs are written with the number of dashes indicating places allotted for input. For example,

100 DAYS   3 HOURS   26 MIN   5 SEC

would be written as follows for a format DAYS HOURS . MIN SEC

1 0 0 0 3 . 2 6 0 5

number is stored in the cell immediately preceding the cell where the last number was stored. Thus, an entire array may be read in by specifying the initial location only.

e. Conversion Codes. There are three codes used in FINP in the program. They are G, Blank and E. G is placed in Column No. 1 of the first FINP card. The variable or array name in first location field on the first card is saved and if data cards follow with blank location fields the corresponding data is stored consecutively in descending order beginning with the cell specified in the first location field on the G Card. Columns 7 to 72 are ignored on the G Card and may be used as a comment card to identify the run.

A blank in the conversion code field indicates that the number in the value field times the power of ten in the exponent field is converted to floating point binary. All the FINP data in the program uses the blank conversion code. The E is placed in the conversion code field following the last data field being used. This defines an end-of-case and control is returned to the program. The rest of this field and the remaining fields on the card are ignored.

An example of a set of FINP cards is presented in Figure 2-2.

```
GS    TEST CASE FOR GODDARD
100   1.35         151  6.28         GUID 1.           GID  2.
PI    1.00              1.00         2.00                   3.
C1OMP1.           END DATA
```

Figure 2-2 FINP Input Data

The FINP cards for the first run are placed immediately behind the program data card. If a series of runs are stacked, the second set of FINP cards is placed behind the GOTOB cards for the first run. The FINP and GOTOB card sets are alternated with FINP cards being first.

2.4 GOTOB INPUT DATA

The GOTOB data cards are used to control the type of tracking being used at any point on the trajectory. The cards control and can change the types of measurements being made and number stations being used or bodies being observed anywhere along the trajectory. The times to call MATSUB and the times for output are also controlled by the GOTOB cards. The GOTOB cards which control the call of MATSUB (see Figure 2-1) must be filled out carefully so that it will be called at reasonable intervals.

The selection of times to call MATSUB must be made so the vehicle doesn't pass over a tracking station of interest without the station making any observations (see Figure 2-3). If tracking station, moon beacon or onboard measurements are being made, the quality of the observations are averaged over the interval between MATSUB calls and the covariance matrix is updated accordingly. The first time the station observes the vehicle, no updating of the covariance matrix is done. The second time the vehicle is observed on a given pass, the number of observations, N, for the time interval is computed. The variances of the measurements are then divided by N to yield the effect of N measurements on the knowledge of state covariance matrix. Therefore, MATSUB must be called regularly enough to provide good averaging over a station pass. Finally, a guidance correction can only be made after a MATSUB call; therefore, MATSUB must also be called at times when guidance corrections are desired.

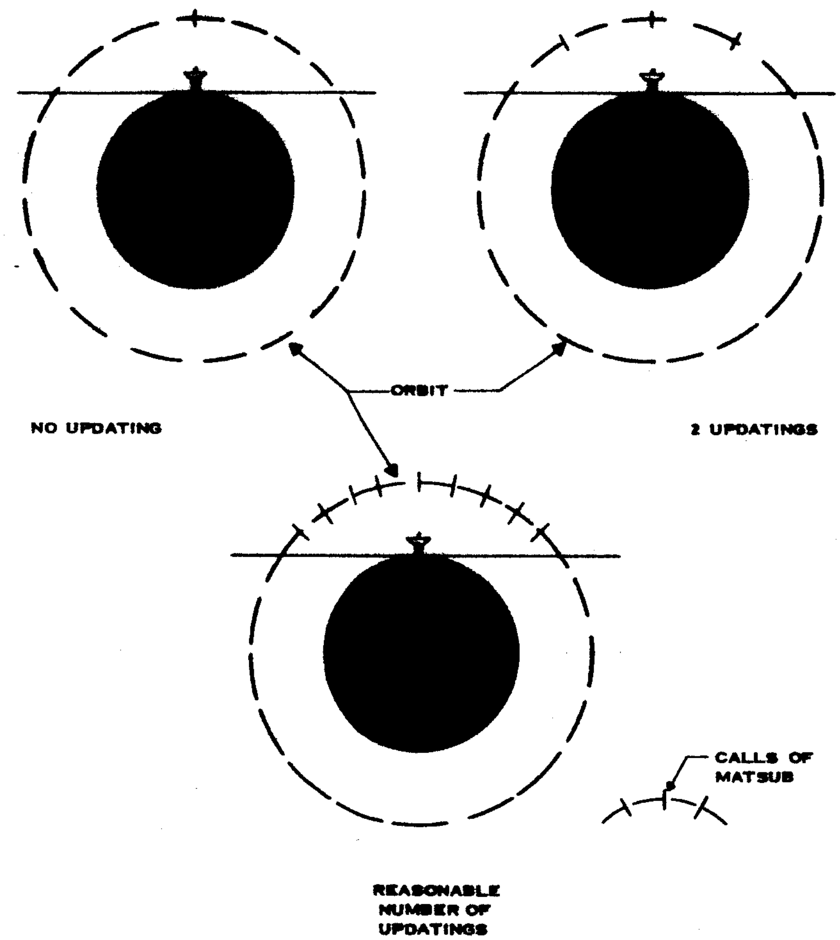

Figure 2-3 Effect of Interval of Calling MATSUB

2.5 GOTOB DATA CARD FORMAT

There are a pair of GOTOB data cards which make up the type of run being made over a specified time interval along the trajectory. The time interval may be long or short and changed as often as desired. Each phase of the trajectory is controlled by a pair of GOTOB cards. The GOTOB card pairs are stacked behind the FINP data cards for the run.

The first card of the pair is a card which is coded to indicate the tracking stations, celestial bodies and moon beacons being used for the particular phase. It also indicates the types of measurements being made during the specified phase of the trajectory. The card is shown in Figure 2-4.

The first 40 columns are divided into 20 pairs of two columns each. One pair is allotted to each of the 20 tracking stations. Then there are four spaces followed by six pairs of two columns each for celestial bodies used with onboard measurements. The 12 onboard spaces are followed by four spaces and then the final 20 spaces are grouped into ten pairs of two columns for use with the moon beacons. If any of the tracking stations, on-board measurements or moon beacons are to be used, a number from 1 to 15 is placed in the appropriate columns to select the particular station, beacon or body.

If no observations are being made during a particular phase, a blank card must be used for the first GOTOB card. The selection of the appropriate number, 1 to 15, to be used in setting up the measurement combination is based on the codes shown in Figure 2-4. The decimal number is converted to binary in the program and used to select the appropriate type of measurement combination. As an example, if range and AZEL were to be measured by earth tracker number seven and range rate, RA, and DEC of moon beacon number two, the first GOTOB card would be filled out as follows (see Figure 2-5). A "3" would be placed in column 14 for the earth based tracking and a 14 starting in column 63 for the moon beacon. The columns are selected because they represent

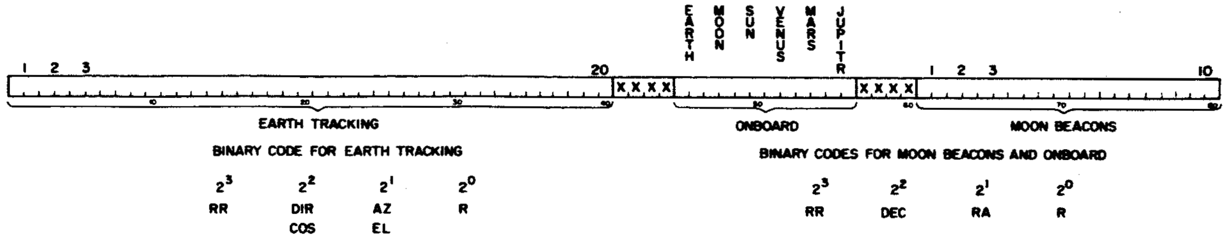

Figure 2-4 Station GOTOB Data Card

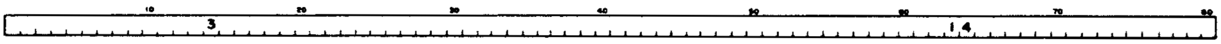

Figure 2-5 Example GOTOB Data Card

tracker number seven and moon beacon number two. The numbers 3 and 14 were selected to describe the types of measurements being made. The numbers 3 and 14 are converted into binary numbers which have the following measurement codes:

Earth Tracker		Moon Beacon	
DEC	Binary	DEC	Binary
3	0 0 1 1	14	1 1 1 0
	AZ EL, R		RR DEC RA

The measurement numbers for a station are input in fixed point. Numbers less than ten are placed in the right column of the station, beacon, or body pair of columns.

The second GOTOB data card for a phase is a card which sets up the calling sequence for MATSUB. An example of the card is presented in Figure 2-6.

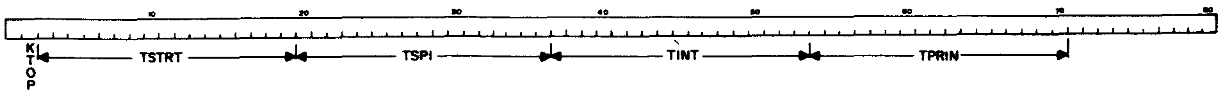

Figure 2-6 MATSUB GOTOB Data Card

A fixed point number 0 or 1 is placed in Column 2. The number is KTOP which if 0 indicates there are additional phases going to be used along the trajectory and additional pairs of GOTOB cards follow the pair being read. The final pair of a set of GOTOB cards for a given trajectory are so indicated by a 1 being placed in Column 2. The remainder of the card is divided into four fields of 17 columns. The fields start in Columns 3, 20, 37 and 54. These fields contain the time to start calling MATSUB, time to stop calling MATSUB, interval of calling MATSUB and print interval respectively. The numbers are input in floating point with the following format:

DAYS HOURS . MIN SEC

The GOTOB cards for the first phase of the trajectory phase are placed immediately behind the FINP cards for the run with the station card first and the card with MATSUB call times second. If more than one set of phase cards is used, the second and succeeding sets are placed immediately behind the first pair of cards. The last MATSUB calling card of the sequence must have a 1 in Column 2 for KTOP. This indicates that the data for all the phases of the run have been read into the program. If runs have stacked, the second set of FINP cards will follow the GOTOB card which has KTOP (Column 2) set equal to one.

SECTION 3

SETUP OF PROGRAM FOR DATA RUN

3.1 ORBIT DETERMINATION

The following is a description of the setup of an error propagation run for tracking system evaluation on the basis of orbit determination. The data would be input through FINP as described earlier (see Table 2-1); TIM, CIBDY, TIBDY, SKTB, TSTOP, X, VX, DATE, FDATE, PI, PUPIN, and OUTTP. These quantities are sufficient to setup the initial knowledge of state covariance matrix and nominal trajectory. The only additional data required to perform the error propagation run is the data (variances) on the measurements which are to be used. These quantities are stored in the S array. Many of these quantities are placed in the S array through the main program calling subroutine CONSTI. If any desired quantities in S are missing (see listing of subroutine CONSTI) or if a different number is desired, they are read in through FINP by using location S. This option allows any of the quantities in the S array to be input or to overlay any numbers which are there.

Subroutine CONSTI has a number of constants compiled in it which should be checked when a run is made. The following stations are presently in CONSTI and have error data associated with them.

Station Number	Station
1	Antigua Radar
2	Ascension Radar
3	Millstone Hill Radar
4	Mobile
5	AMR Tracker
6	Bermuda
7	Goldstone Receiver
8	Goldstone Transmitter
9	GBI Radar

Station Number	Station
10	Johannesburg
11	Hawaii
12	Jodrell Bank
13	Puerto Rico Radar
14	San Salvador
15	Woomera
16	JPL Cape Canaveral
17	Majunga
18	Carnarvon
19	Rosman

In order to establish the location of the variances for the various measurement errors in the S array, a list is presented in Table 3-1 which describes the location of these quantities.

If the knowledge of state covariance matrix is being input in launch pad tangent plane the following quantities must be placed in S.

S(80) = LAUNCH PAD LATITUDE	DEG
S(81) = LAUNCH PAD LONGITUDE	DEG
S(82) = LAUNCH PAD ALTITUDE	KM
S(83) = TIME FROM LAUNCH TO INJECTION	DAYS HRS . MIN SEC
S(84) = FIRING AZIMUTH	DEG

After the stations to be used and the kinds of measurements to be made have been selected, the appropriate errors are placed in the S array through FINP. The GOTOB input cards are then set up to: 1) select these stations and measurements, and 2) call MATSUB at reasonable time intervals. An example test case for this type of run is carried through and the results presented at the end of this document.

TABLE 3-1

EARTH BASED TRACKING DATA

N = 0, 19 where N + 1 is the Tracking Station Number

S(125) + N * 15	Variance of Range	KM^2
S(126) + N * 15	Variance of Azimuth	RAD^2
S(127) + N * 15	Variance of Elevation	RAD^2
S(128) + N * 15	Variance of Range Rate	$(KM/SEC)^2$
S(129) + N * 15	Variance of Latitude	RAD^2
S(130) + N * 15	Variance of Longitude	RAD^2
S(131) + N * 15	Variance of Altitude	KM^2
S(132) + N * 15	Variance of Azimuth Bias	RAD^2
S(133) + N * 15	Variance of Elevation Bias	RAD^2
S(134) + N * 15	Latitude of Station	DEG
S(135) + N * 15	Longitude of Station	DEG
S(136) + N * 15	Altitude of Station	KM
S(137) + N * 15	Station Name	
S(138) + N * 15	Period of Observation	SEC
S(139) + N * 15	Variance of Time Bias	SEC^2
LAST CELL S(424)		
S(788) + N * 5	Station Artificial Horizon Height	RAD
S(789) + N * 5	Variance of (M) Direction Cosine	
S(790) + N * 5	Variance of (M) Direction Cosine Bias	
S(791) + N * 5	Variance of (L) Direction Cosine	
S(792) + N * 5	Variance of (L) Direction Cosine Bias	
LAST CELL S(887)		

TABLE 3-1 (Continued)

ONBOARD TRACKING DATA

S(426)	Variance of Range	KM^2
S(427)	Variance of Right Ascension	RAD^2
S(428)	Variance of Declination	RAD^2
S(429)	Variance of Range Rate	$(KM/SEC)^2$
S(430)	Variance of Range Bias	KM^2
S(431)	Variance of Right Ascension Bias	RAD^2
S(432)	Variance of Declination Bias	RAD^2
S(433)	Variance of Range Rate Bias	$(KM/SEC)^2$
S(434)	Variance of Clock Time Bias	SEC^2
S(435)	Spare	
S(436)	Spare	
N = 0, 5	Earth, Moon, Sun, Venus, Mars, Jupiter	
S(437) + N * 4	Period of Range Observation	SEC
S(438) + N * 4	Period of Right Ascension Observation	SEC
S(439) + N * 4	Period of Declination Observation	SEC
S(440) + N * 4	Period of Range Rate Observation	SEC
LAST CELL S(456)		

TABLE 3-1 (Concluded)

LUNAR BASED BEACONS TRACKING DATA

N = 0, 9 where N + 1 is Lunar Beacon Number

S(500) + N * 15	Variance of Range	KM^2
S(501) + N * 15	Variance of Right Ascension	RAD^2
S(502) + N * 15	Variance of Declination	RAD^2
S(503) + N * 15	Variance of Range Rate	$(KM/SEC)^2$
S(504) + N * 15	Variance of Range Bias	RAD^2
S(506) + N * 15	Variance of Declination Bias	RAD^2
S(507) + N * 15	Variance of Range Rate Bias	$(KM/SEC)^2$
S(508) + N * 15	Variance of Clock Bias	SEC^2
S(509) + N * 15	Latitude of Station	DEG
S(510) + N * 15	Longitude of Station	DEG
S(511) + N * 15	Altitude of Station	KM
S(512) + N * 15	Station Name	
S(513) + N * 15	Period of Observation	SEC
S(514) + N * 15	Spare	
LAST CELL S(649)		
S(758) + N * 3	Variance of Latitude	RAD^2
S(759) + N * 3	Variance of Longitude	RAD^2
S(760) + N * 3	Variance of Altitude	KM^2
LAST CELL S(787)		

3.2 GUIDANCE RUN

The following is a description of the setup of an error propagation run for guidance analysis. There are two types of guidance laws which may be used in the program. The number which is input for GUID in FINP selects either Fixed Time of Arrival or Constant Energy with Respect to the Target. In order to perform a reasonable guidance run, it must be done in conjunction with some type of observations to determine the state of the vehicle. Therefore, the first part of the guidance run setup is the same as the run previously described for evaluation of a tracking system. In addition to the setup of a tracking system, the following guidance data must be read in through FINP (see Table 2-1); GUID, CIOMP, TGUID, GID, PARI, and PARIN. If the initial deviations from nominal covariance matrix is input in launch pad coordinates, the cells S(80) to S(84) must be input as described in the previous section.

There are two additional options which can alter the above inputs. If CIOMP is set positive, the CTE (Constant Energy) and FTA (Fixed Time of Arrival) guidance partial matrices are computed. If these matrices are available, they may be read in through FINP and CIOMP may be neglected and the matrices will not be computed. RETR may also be set one and then the program will assume the vehicle is at perigee at the point of stopping and carry the knowledge of state and deviation from nominal covariance matrices through a RETRO maneuver into circular orbit.

If the GUID, CIOMP, and RETR options are desired on each of successive stacked runs, they must be input for each run since they are set zero following each run.

Guidance error quantities which must be input in the S array are the following:

S(471) = Rocket Motor Shutoff Error $(\%)^2$
S(472) = Rocket Motor Pointing Error $(\mathrm{RAD})^2$
S(484) = Error in Monitoring Guidance Correction $(\%)^2$

The quantity in S(484) is used in updating the knowledge of state covariance matrix for the lack of knowledge in the guidance correction. The use is shown below

$$P_A = P_B + S(484) \begin{bmatrix} 0 & 0 \\ 0 & E(qq^T) \end{bmatrix}$$

Where $E(qq^T)$ is the covariance matrix of the error in the guidance correction.

The above setup will provide guidance corrections at times specified by TGUID. There is an additional option which may be exercised by placing a one in S(475). The program will then make guidance corrections at times when the following test is satisfied.

$$\frac{\text{RMS Knowledge of Miss}}{\text{RMS Miss}} - \text{CORR} \leq 0$$

The quantity called CORR is stored in S(483) and when this type of guidance is selected, must be input through FINP. It must be remembered that the subroutine which performs the guidance function (GUID) along the trajectory is called by MATSUB. The GOTOB cards must, therefore, be set up to call MATSUB when guidance corrections are desired.

SECTION 4

EXAMPLE TEST CASE

4.1 INTRODUCTION

In order to determine an adequate navigation system for a space mission, a large number of parametric studies are required. A few of the possible parametric studies have been performed here for a lunar mission. The purpose of these runs is not to perform a detailed guidance analysis for the mission but to demonstrate some of the capabilities of the error propagation program. The test case results will answer a few of the many questions which one could ask concerning the guidance system (including orbit determination) for such a lunar mission. The covariance matrix which was used for the initial knowledge of state and deviations from the nominal was an arbitrary selection. The coordinates are injection tangent plane with the following order:

$$\frac{\vec{R} \times \vec{v}}{|R \times v|} \times \frac{\vec{R}}{|R|};\ \frac{\vec{R} \times \vec{v}}{|R \times v|};\ \frac{\vec{R}}{|R|};\ \text{and corresponding velocities.}$$

$$PARI = PI = \begin{bmatrix} 30. & 0 & -20. & -.025 & 0 & -.029 \\ & .64 & 0 & 0 & 3.0\cdot 10^{-3} & 0 \\ & & 15. & .019 & 0 & .023 \\ & & & 2.5\cdot 10^{-5} & 0 & 3.3\cdot 10^{-5} \\ & & & & 1.8\cdot 10^{-5} & 0 \\ & & & & & 4.6\cdot 10^{-5} \end{bmatrix}$$

4.2 NOMINAL TRAJECTORY

The mission requirements which were considered in obtaining the initial conditions for the nominal trajectory from a patch conic program were the following:

a. 72-hour flight time

b. Equatorial lunar orbit

c. Lunar orbit altitude of 100 nautical miles

d. Earth parking orbit altitude of 100 nautical miles.

The nominal trajectory, which was selected from refined patch conic conditions for use in the test case, had the following characteristics:

Geocentric Characteristics of Transfer Ellipse at Injection

Date and time of injection in earth-moon coast ellipse	2 January 1965 $5^h 1^m 25.33^S$
Inertial velocity at injection	10.89 KM/SEC
Latitude of injection	10.89 degrees
Longitude of injection	326.39 degrees
Flight path angle	.0929 degrees
Altitude at injection	272.17 KM
Time from injection to perilune	$71^h 21^m 35^S$
Semi-major axis	318,403 KM
Eccentricity	.97911
Inclination	28.18 degrees

Lunar Arrival Conditions (Selenographic Coordinates)

Date and time at perilune	5 January 1965 $4^h 23^m 0.321^S$
Inclination of trajectory plane	179.47 degrees
Line of nodes	224.40 degrees
Altitude of perilune	183.82 KM
Velocity of perilune	2.421 KM/SEC
Latitude of perilune	.359 degrees
Longitude of perilune	181.93 degrees
Argument of perilune	42.47 degrees

This nominal trajectory has the feature that perilune is occulted from the earth by the moon. It is not possible, therefore, to observe a retro maneuver at perilune. (This effect is printed out in the program as part of the station geometry.) The trajectory is presented in Figures 4-1 through 4-4 in Cartesian equator of data coordinates.

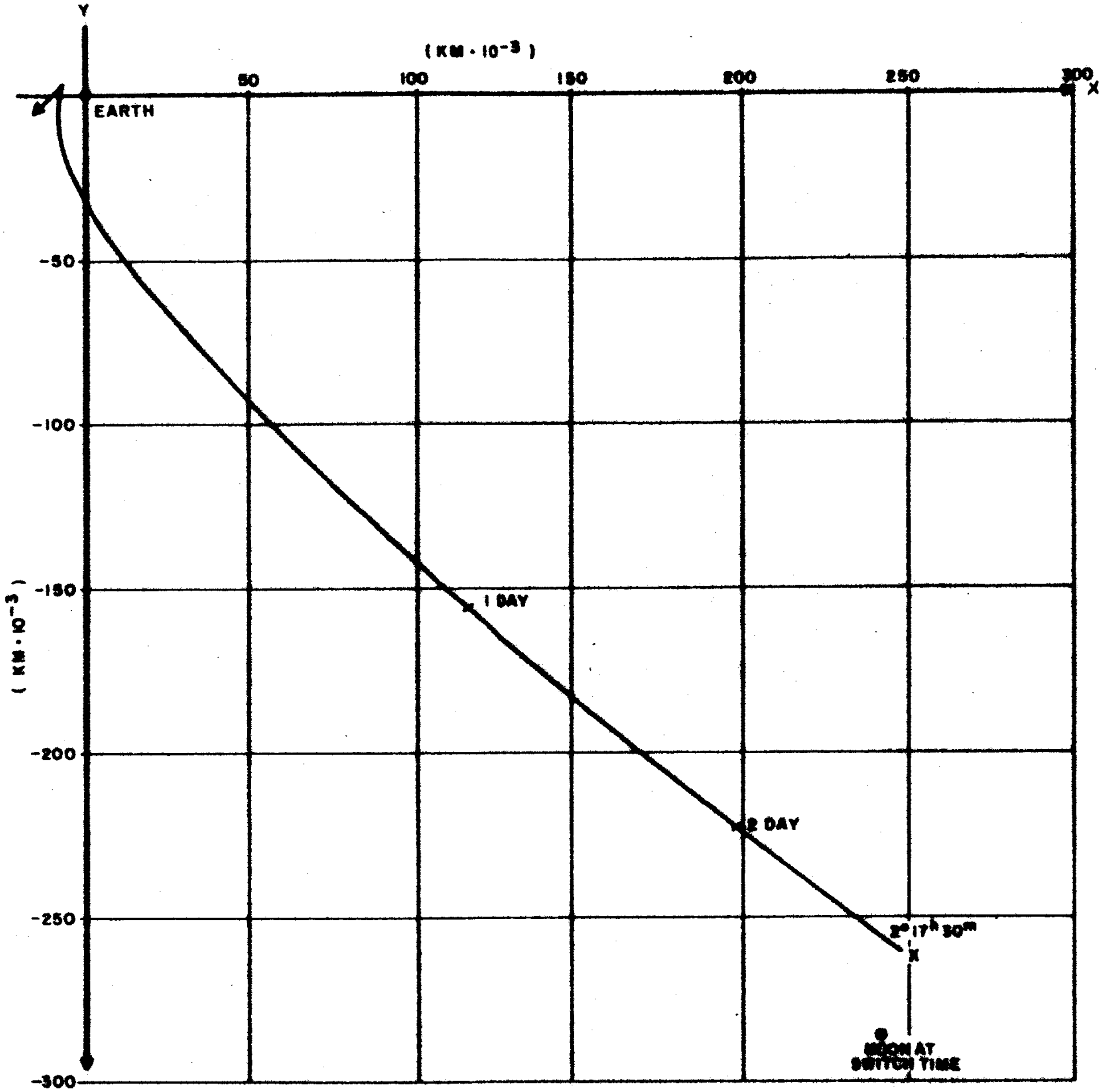

Figure 4-1 Nominal Trajectory Earth Centered
Equator Date X-Y Plane

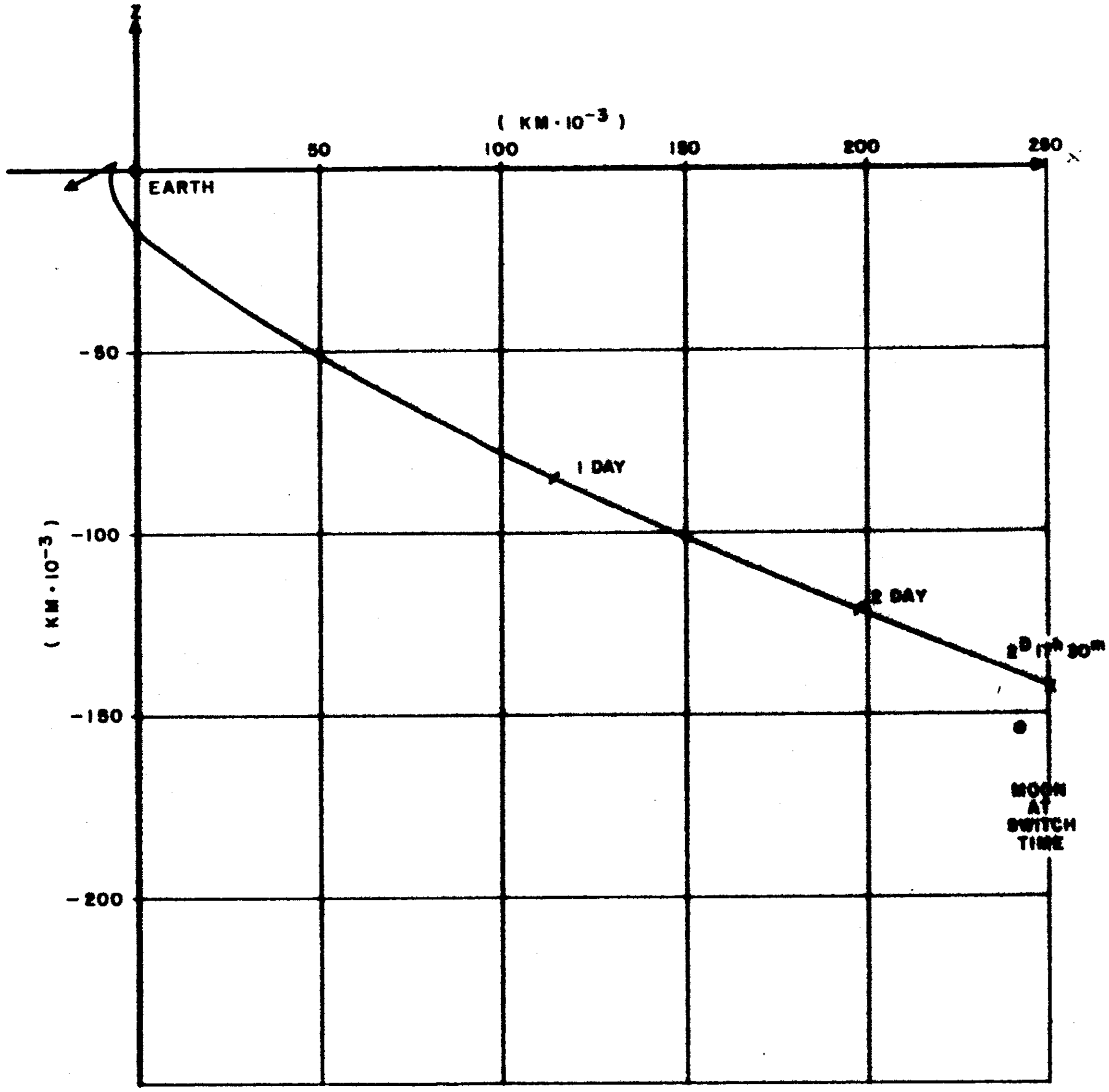

Figure 4-2 Nominal Trajectory Earth Centered Equator Date X-Z Plane

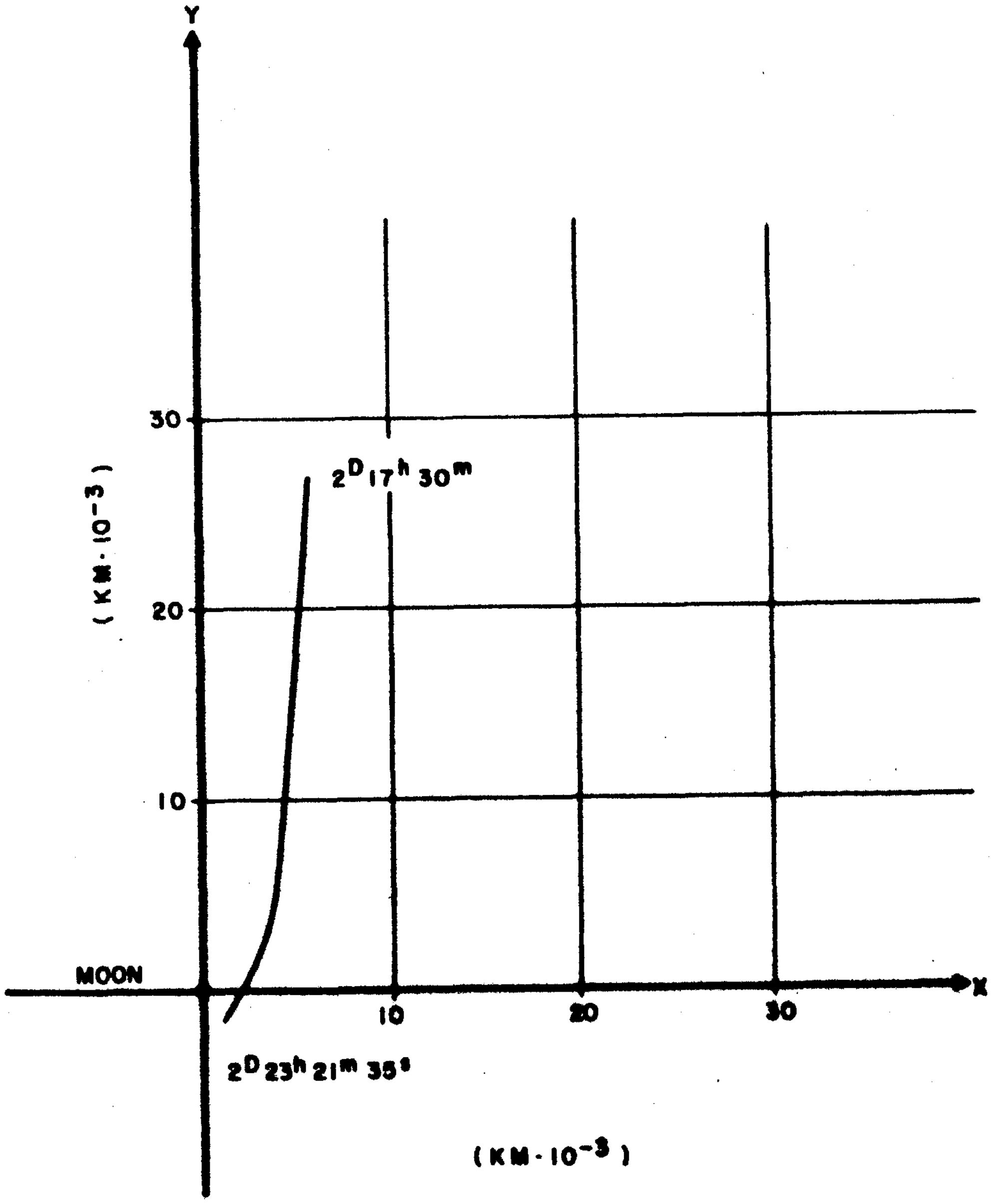

Figure 4-3 Nominal Trajectory Moon Centered
Equator Date X-Y Plane

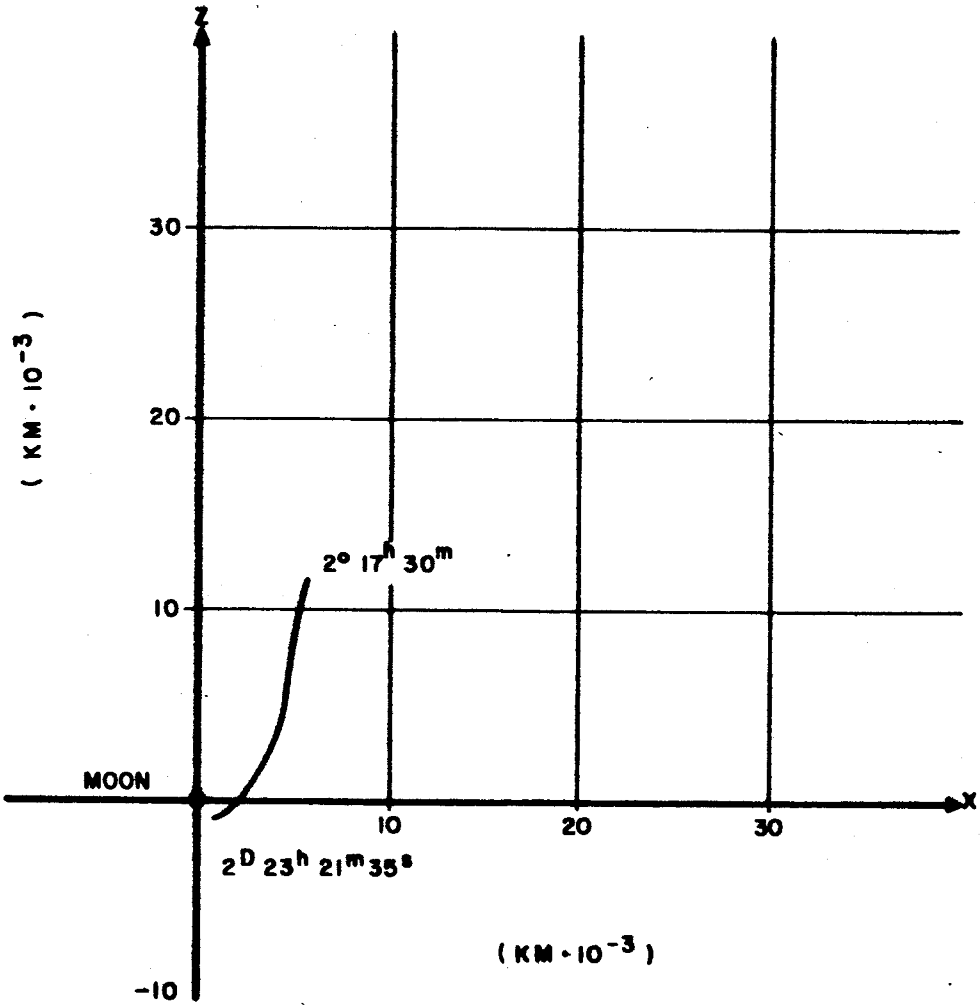

Figure 4-4 Nominal Trajectory Moon Centered Equator Date X-Z Plane

4.3 SETUP OF INPUT DATA

The setup of the FINP cards and GOTOB cards will be described in detail for one of the test case runs. Assume we desire to make a guidande run with two corrections using a FTA (Fixed Time of Arrival) guidance law and the DSIF tracking net.

The FINP cards would look like those presented in Figure 4-5. The format of the input has been described in Section 2.3. The information which is on each card will be described here to indicate its meaning.

Card 1 The G indicates a temporary storage location for the "S" array. Beyond Column 6, the card can be used for a comment card.

Card 2 The numbers in the location fields are cells of the "S" array identified on the first card. The locations represent station data which are being input. The remainder of the desired station data is already in CONSTI and doesn't need to be read in.

Card 3 The first two values are more numbers in the "S" array. The third location on the card has a variable name in it which is the injection date. This also stops the "S" array input. The value represents the following date: January 2, 1965. The final location on the card is the fractional date. The value represents $5^h1^m25.33^S$.

Card 4 This card contains four variables which represent: 1) type of stop, time only; 2) type of input coordinates, Cartesian equator of date; 3) central body, earth; and 4) target body, moon.

```
GS    TEST RUN FTA GUIDANCE DSIF TRACKING
 818  .087         833  .087         858  .087         228  60.
 273  60.          348  60.          DATE 6501.02      FDATE501.2533
 SKTB 3.           TIM  2.           CIBDY1.           TIBDY2.
 PUPIN2.           PARIN2.           TSTOP223.2135     GUID 1.
 CIOMP1.           GID  2.           RETR 1.           PARI 1.
 7    1.           12   1.           16   1.           19   1.
 21   1.           P1   1.           7    1.           12   1.
 16   1.           19   1.           21   1.           X        -5072.7470
      3771.6547          2066.1332   VX   -7.0559528        -7.3600335
      -3.8678243   TGUID12.               212.        END DATA
```

Figure 4-5 FINP Data Cards

Card 5 This card contains four variables which represent: 1) type of PI matrix coordinates, injection tangent plane; 2) type of PARI matrix coordinates, injection tangent plane; 3) time to stop run, 2 days, 23 hours, 21 minutes and 35 seconds; 4) type of guidance, Fixed Time of Arrival.

Card 6 This card contains three variables and the start of an array. The variables express the following: 1) guidance matrices are to be computed; 2) there are to be two guidance corrections; 3) perform a retro-maneuver at the end time. The final location on the card is PARI which starts the input of the deviation from nominal covariance matrix. The value represents the first cell of the array.

Card 7 This card contains the 7, 12, 16 and 19 elements of the PARI array.

Card 8 This card contains the 21 element of the PARI array. Location 2 on the card starts the input of the PI array which is the knowledge of state covariance matrix. The 1, 7, and 12 elements are on the card.

Card 9 This card contains the 16, 19 and 21 elements of the PI array. The final location is the start of the injection position vector. The value contains the x coordinate.

Card 10 The first two locations contain the y and z coordinates of the injection position. The third location starts the injection velocity coordinates. The value in the third value field is the x velocity. The last field contains the y velocity.

Card 11 The first field contains the z velocity component. The second location is the start of the array for guidance correction times. The second and third fields contain the following times for correction: 12 hours and 2 days, 12 hours respectively.

The second set of input cards which must be set up are the GOTOB cards. The DSIF stations which will be used are: Goldstone, Johannesburg and Woomera. The types of measurements being made by each are: range rate, azimuth and elevation. The stations being used have much of the desired station error data compiled in CONSTI. Therefore, the stations are assigned the station numbers which correspond to the location of errors in CONSTI. These numbers are: No. 7, Goldstone; No. 10, Johannesburg; and No. 15, Woomera. The number code which represents the desired measurement combination is obtained as follows:

RR	DIR COS	AZ EL	RANGE
2^3	2^2	2^1	2^0

RR, AZ, and EL = 8 + 2 = 10

The above data is used on the first card of the pair of GOTOB cards. The card is shown in Figure 4-6.

Figure 4-6 Station GOTOB Data Card

The second GOTOB card is to set up the MATSUB calling sequence. The trajectory will be divided into the following six phases.

1. Start Time = 0; Stop Time = 2 hours. Call MATSUB and print out every 10 minutes to obtain good averaging over a station.

2. Start Time = 2 hours; Stop Time = 15 hours. Call MATSUB and print out every 20 minutes to obtain good averaging. This is possible since station geometry is changing more slowly. The call sequence also ensures the guidance correction can be made at 12 hours.

3. Start Time = 15 hours; Stop Time = 2 days, 10 hours. Call MATSUB and print out every 30 minutes, vehicle is well out in midcourse flight.

4. Start Time = 2 days, 10 hours; Stop Time = 2 days, 15 hours. Call MATSUB and print out every 20 minutes, ensures calling MATSUB for guidance correction at 2 days, 12 hours and data smoothing following correction.

5. Start Time = 2 days, 15 hours; Stop Time = 2 days, 20 hours. Call MATSUB and print out every 30 minutes, vehicle is well out along trajectory.

6. Start Time = 2 days, 20 hours; Stop Time = 2 days, 24 hours. Call MATSUB and print out every 10 minutes, vehicle is coming into target and errors are changing rapidly.

The complete set of GOTOB data cards for the above described run is presented below in Figure 4-7. These cards are placed immediately behind the FINP cards shown in Figure 4-5 for this data run. The combined set of data will then perform the desired guidance run to the moon.

```
          10   10        10
00.            2.               .10               .10
          10   10        10
02.            15.              .20               .20
          10   10        10
015.           210.             .30               .30
          10   10        10
0210.          215.             .20               .20
          10   10        10
0215.          220.             .30               .30
          10   10        10
1220.          224.             .10               .10
```

Figure 4-7 GOTOB Data Cards

The setup procedure described above was used to obtain the data to be presented for the test case.

4.4 ORBIT DETERMINATION

The test case nominal trajectory was run using four different tracking and observation systems. The four systems are described in Table 4-1. The DSIF tracking system consisted of the following stations measuring range rate, azimuth and elevation: Goldstone, Woomera and Johannesburg. The system, which was TESTNET, consisted of the following stations measuring range, range rate, azimuth and elevation: Rosman, Carnarvon and Majunga. The onboard systems are described in the Table. The results of the orbit determination of the four systems, DSIF, TESTNET, Onboard, and DSIF with Onboard are presented in Figures 4-8, 4-9 and 4-10. In order to demonstrate the ability to change the tracking system parameters, the DSIF tracking station location errors were removed and a second run made to illustrate the improvement possible by removing station location errors. The effect of this parameter change can be seen by comparing curves (1) and (2) in the figures.

In a similar fashion, it is possible to change any or all of the parameters presented in Table 4-1 and note the effect on the ability of a system to determine an orbit. The system may also be composed of more than one type of measuring device. This was illustrated by adding onboard angle measurements to the DSIF. The result of combining these measurements can be seen by comparing curves (1) and (3) in Figures 4-8, 4-9 and 4-10.

The data which generated the peculiar variations on the curves in Figures 4-8 and 4-10 were examined to determine the origin of the variations. An explanation of these variations is presented below.

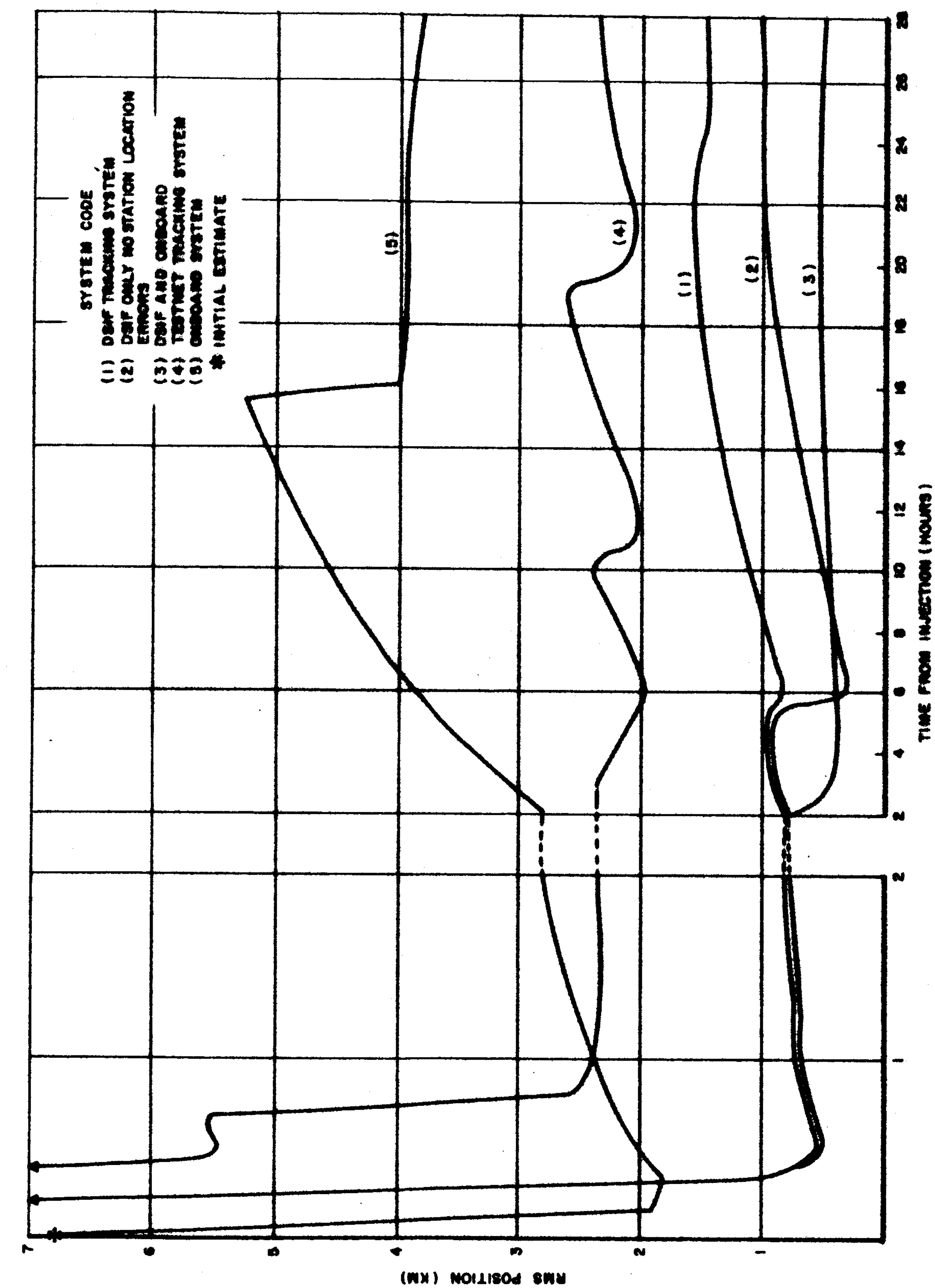

Figure 4-8 RMS Knowledge of Position

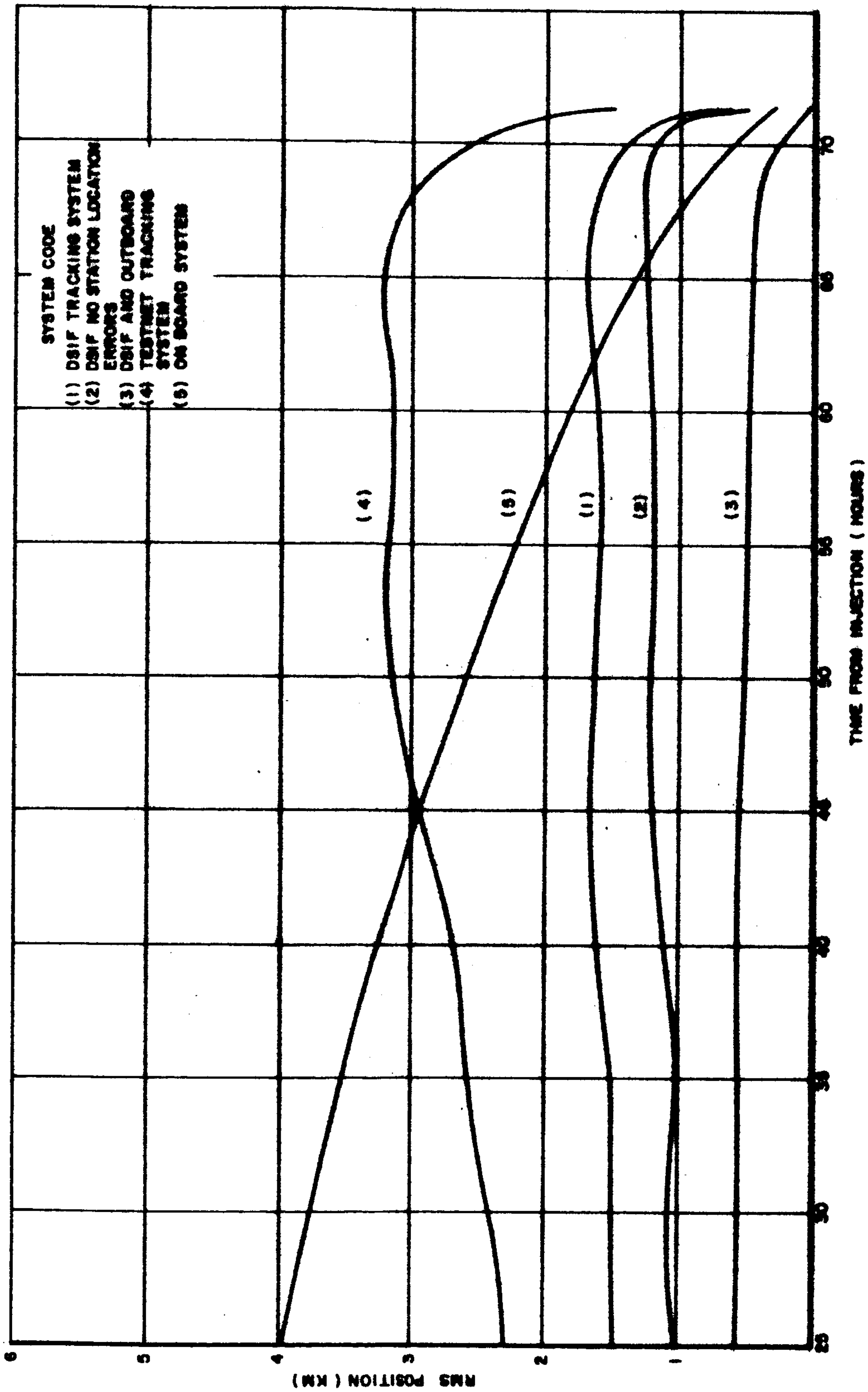

Figure 4-9 RMS Knowledge of Position

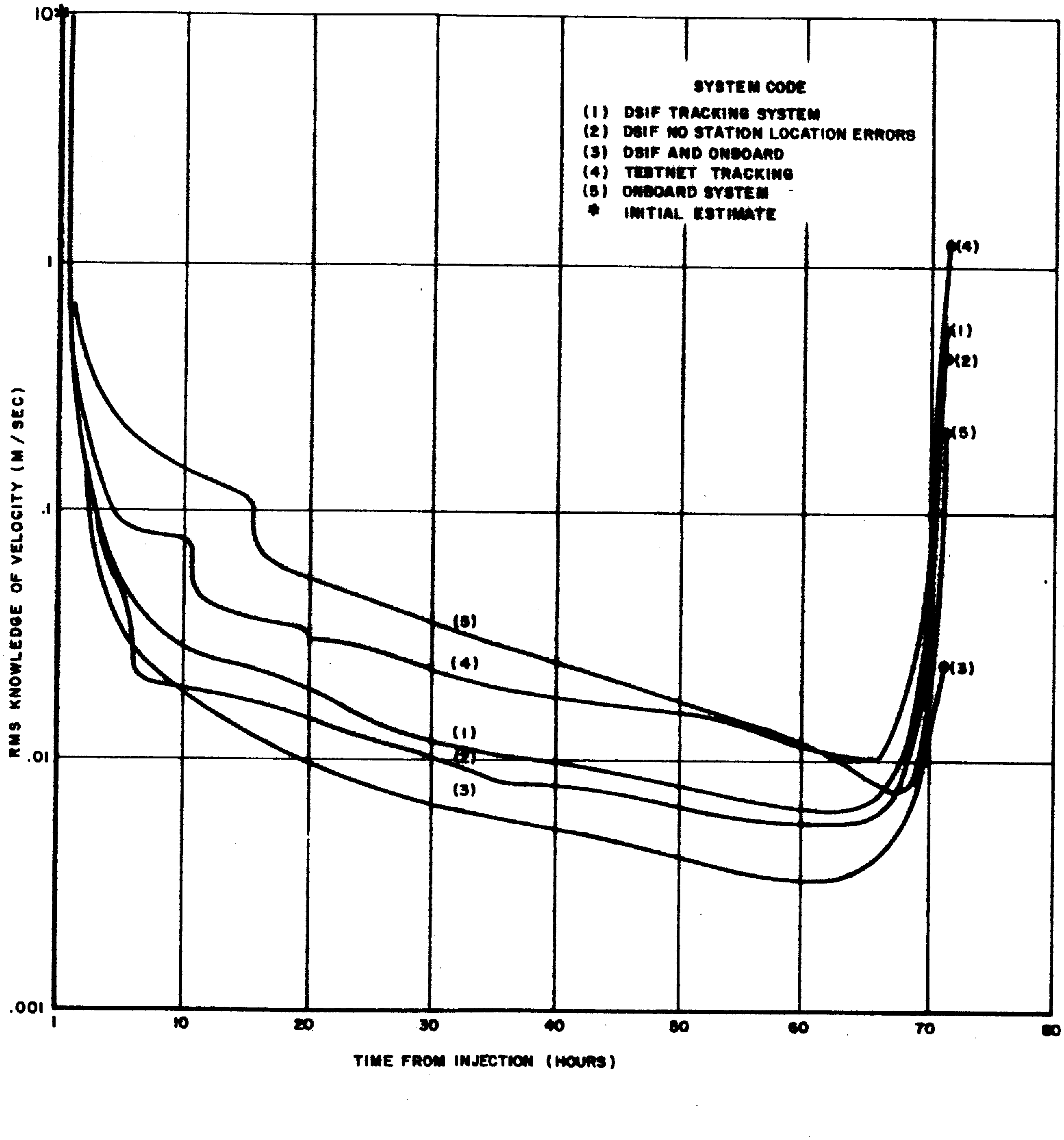

Figure 4-10 RMS Knowledge of Velocity

TABLE 4-1

TRACKING SYSTEM USED IN STUDY

	TESTNET Standard Deviation	DSIF Standard Deviation
Range	200 meters	
Range Rate	2 m/sec	.2 m/sec
Azimuth/Elevation	20 mills	.2 mills
Horizon	5 degrees	5 degrees
Observation Period	20 seconds	60 seconds
Latitude Error	63 meters	63 meters
Longitude Error	63 meters	63 meters
Altitude Error	10 meters	3 meters

ONBOARD SYSTEM

	Body(1)	Standard Deviation	Period of Observation
Right Ascension	(E,M,S)	.17 mills	600 seconds
Declination	(E,M,S)	.17 mills	600 seconds
Range Rate	(E)	2 m/sec	60 seconds
Right Ascension	(MB)	200 mills	60 seconds
Declination	(MB)	200 mills	60 seconds
Range Rate	(MB)	2 m/sec	60 seconds
Latitude Error	(MB)	1 KM	
Longitude Error	(MB)	1 KM	
Altitude Error	(MB)	1 KM	

DSIF ONBOARD SYSTEM

Right Ascension	(E,M,S)	.017 mills	600 seconds
Declination	(E,M,S)	.017 mills	600 seconds

(1) (E,M,S) = Earth, Moon, Sun; (MB) = Moon Beacon; (E) = Earth

a. Curve (1) and (2). On curves (1) and (2) for the DSIF at approximately six hours, the Johannesburg tracker was in the orbital plane (elevation 88.6 degrees) and was able to reduce the position and velocity errors in the in-plane coordinate, $\hat{v} \times (\hat{R} \times \hat{V})$. The errors in this coordinate, at the time, were the largest in the knowledge of the state covariance matrix.

b. Curve (3). The improvement in the knowledge of state at two hours is due to the addition of the onboard measurements to the DSIF at that time.

c. Curve (4). The periodic variations which occur in this run are the result of the number of stations which are observing the vehicle. Two stations would increase the knowledge of state and as soon as the number of stations observing was reduced to one, the knowledge of state would slowly degrade until two stations could observe it again.

d. Curve (5). The sharp increase in the knowledge of state at approximately 15 hours is due to the addition of moon beacon observations to the onboard system at that time.

4.5 MIDCOURSE GUIDANCE

The midcourse guidance for the test case was performed with two corrections; 12 hours after injection and 60 hours. The times for correction could be optimized in terms of payload, but were not varied for this presentation. The guidance run was made using the DSIF as the tracking network. A run was also made using the tracking system made up of the DSIF and onboard measurements to illustrate the effect of the tracking system on the guidance capability. Two different guidance laws were also used to illustrate the effect of requiring Fixed Time of Arrival as opposed to a guidance law where time is left free to vary. The time constraint was replaced by a constraint on the hyperbolic excess velocity and the position deviation was defined in terms of $\delta(\vec{B}\cdot\hat{T})$ and $\delta(\vec{B}\cdot\hat{R})$(see subroutine BVEC). The guidance law using these constraints is called CTE, Constant Energy with Respect to the Target. The results of these guidance runs are presented in Table 4-2.

The effect of the guidance corrections on the orbit determination accuracy is shown in Figure 4-11. The graph presents a comparison of the DSIF tracking accuracy for runs with and without guidance corrections. The error in monitoring the correction of the guidance run was three percent of the RMS error in making the correction. Figure 4-12 presents position and velocity deviations from the nominal for the FTA guidance law using the DSIF tracking. It also presents the RMS required velocity correction as a function of time.

One final option which was used is that of a retro-maneuver into circular orbit at perilune. The knowledge of position and velocity are shown in Figure 4-13 for the DSIF tracking and FTA guidance law through the retro-maneuver and into circular orbit. The retro-maneuver cannot be observed with earth-based tracking.

4.6 SUMMARY

The results presented in this section represent data for one or two system parameters. In a detailed guidance analysis for a specified mission, the effect of a great many of the various system parameters must be studied to define an optimum system. The error propagation program offers the facility to the user of making these parametric studies on any of the error quantities which have been used in the sample runs. In addition, a large number of possible system configurations for tracking and guidance, compatible with the complexity of the mission, may be evaluated rapidly.

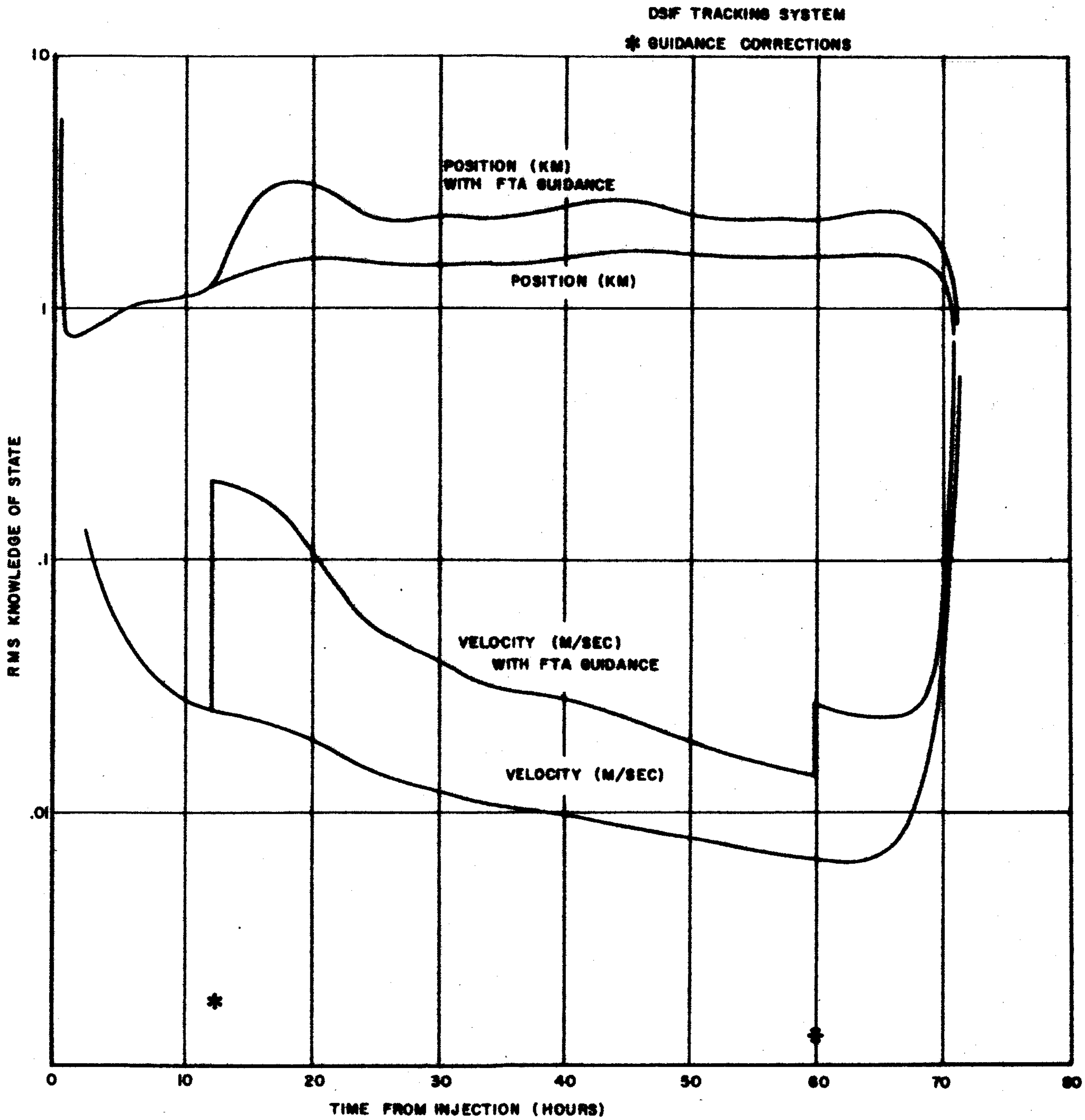

Figure 4-11 RMS Knowledge of Position and Velocity with and without Guidance

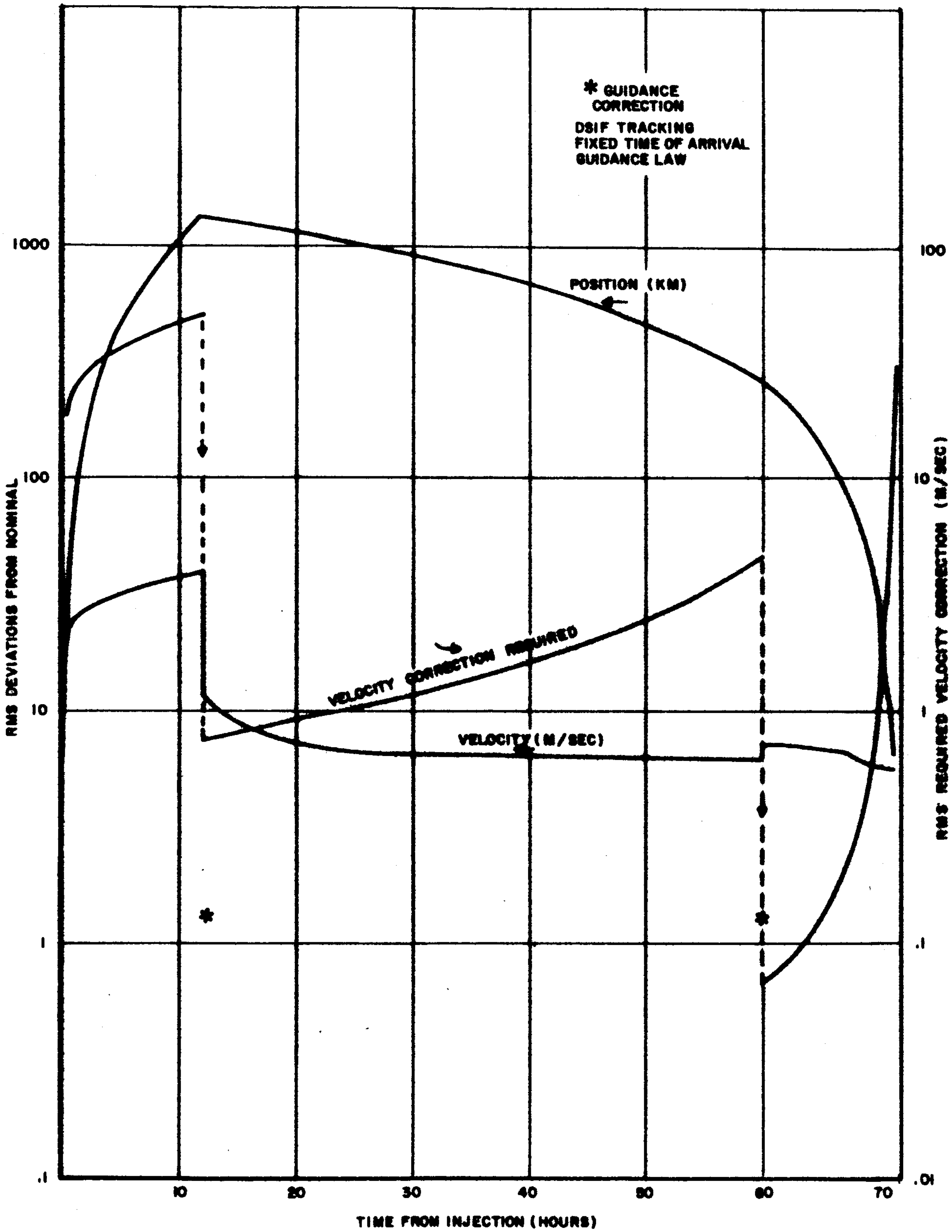

Figure 4-12 RMS Position and Velocity Deviations from the Nominal

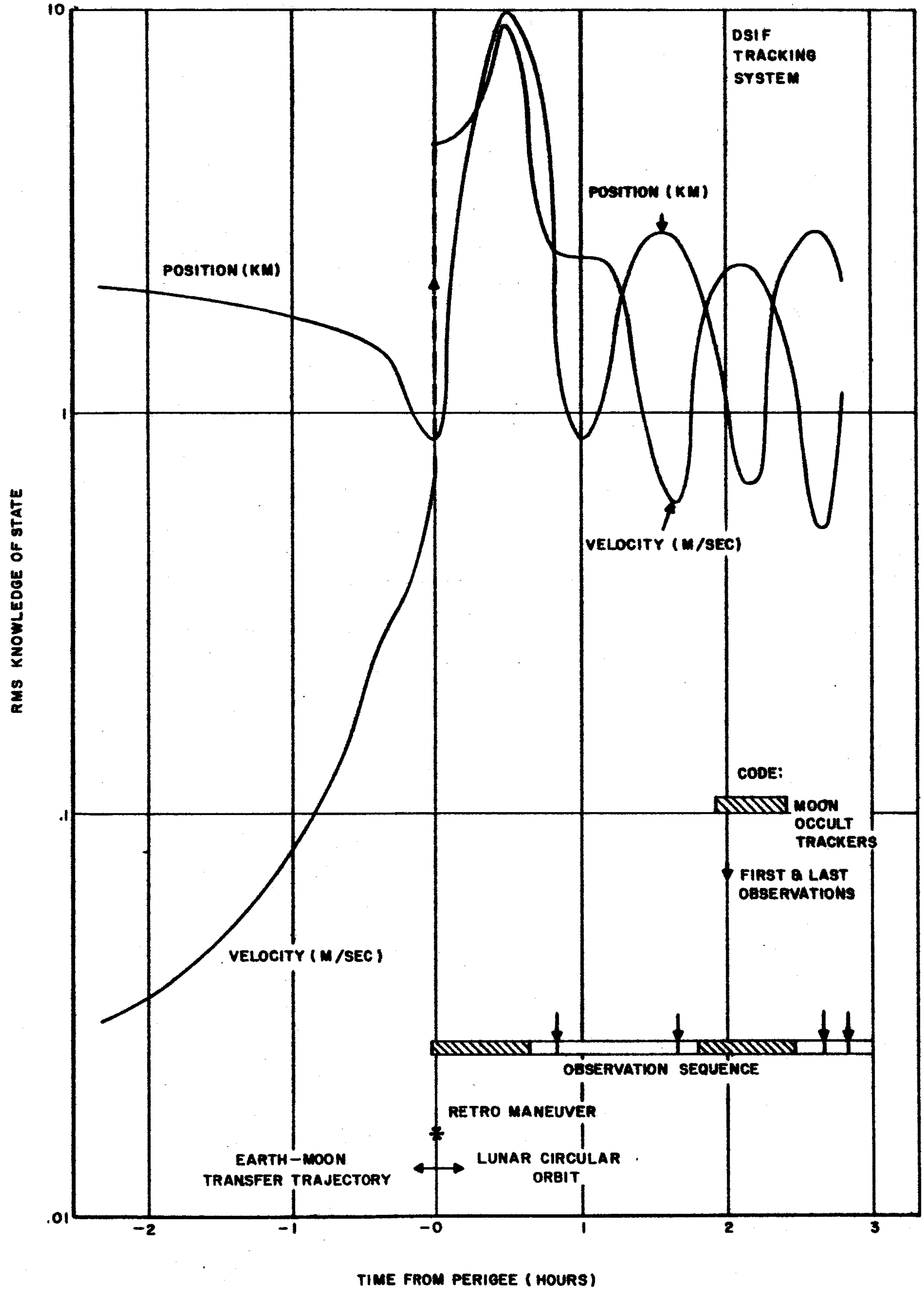

Figure 4-13 RMS Knowledge of Position and Velocity in Lunar Orbit

TABLE 4-2

GUIDANCE DATA

Guidance Correction RMS Error Data

Pointing Error .5° Shutoff Error 1% Error in Monitoring Correction 3%

Tracking System DSIF — Guidance Law CTE

RMS Velocity Correction	RMS Position Deviation at Perilune	RMS V Infinity Dev. at Perilune
Injection	5410 KM	66.8 m/sec
1st Cor. 46 m/sec	126 KM	.673 m/sec
2nd Cor. 2.72 m/sec	2.12 KM	.0266 m/sec

Tracking System DSIF — Guidance Law FTA

RMS Velocity Correction	RMS Position Deviation at Perilune
Injection	20095 KM
1st Cor. 51.228209 m/sec	249.6696 KM
2nd Cor. 4.619412 m/sec	6.4621 KM

Tracking System DSIF and Onboard — Guidance Law FTA

RMS Velocity Correction	RMS Position Deviation at Perilune
Injection	20095 KM
1st Cor. 51.228213 m/sec	249.5679 KM
2nd Cor. 4.618891 m/sec	4.3528 KM